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Systematic thermal reduction of neutronization in core-collapse supernovae

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ABSTRACT

We investigate to what extent the temperature dependence of the nuclear symmetry energy can affect the neutronization of the stellar core prior to neutrino trapping during gravitational collapse. To this end, we implement a one-zone simulation to follow the collapse until β -equilibrium is reached and the lepton fraction remains constant. Since the strength of electron capture on the neutron-rich nuclei associated to the supernova scenario is still an open issue, we keep it as a free parameter. We find that the temperature dependence of the symmetry energy consistently yields a small reduction of deleptonization, which corresponds to a systematic effect on the shock wave energetics: the gain in dissociation energy of the shock has a small yet non-negligible value of about 0.4 foe (1 foe = 10^{51} erg) and this result is almost independent from the strength of nuclear electron capture. The presence of such a systematic effect and its robustness under changes of the parameters of the one-zone model are significant enough to justify further investigations with detailed numerical simulations of supernova explosions.

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1. Introduction

Weak interaction processes are naturally associated to core-collapse supernovae and more generally to compact stars. Indeed, on the one hand the increasing density in the collapsing core of a massive star continuously shifts the β -equilibrium conditions and thus drives electron capture (first mostly on exotic nuclei and then on free, unbound protons) all the way to an almost completely deleptonized equilibrium state, the final neutron star. On the other hand, the tremendous densities and temperatures obtained through the gravitational compression allow the neutrinos, produced both thermally and in such weak processes, to interact significantly with matter. The neutrinos diffuse, rather than stream freely, out of the collapsing core and such neutrino transport produces unique physical scenarios, like neutrino trapping half-way in the collapse or the shock-wave revival powered by neutrino cooling of the proto-neutron star. We refer to the review by Hans Bethe [1] for a masterful physical discussion of these phenomena and of their relevance to supernova explosions.

In a previous paper [2] (from now on Paper I; see also Ref. [3] for further details), we discussed a particular issue related to electron capture in collapsing stellar cores. First, we studied the tem-

perature dependence of the nucleon effective mass, m^* , in the nuclei ^{98}Mo , ^{64}Zn and ^{64}Ni as due to the coupling of the mean field single-particle levels to the collective surface vibrations of the nucleus, calculated in the quasi-particle random phase approximation (QRPA). Then, we observed that the decrease with temperature obtained for m^* in the range $0 < T < 2$ MeV induces a corresponding increase of the nuclear symmetry energy (per nucleon), E_{sym} , and in analogy to the results of the Fermi gas model we argued that the temperature dependence of E_{sym} can be fitted by a simple analytical expression. Finally, we investigated the implications of such a temperature dependence on the gravitational collapse of the core of massive stars. We did this in a one-zone (uniform mean density) model, an approach which incorporates the important physics but is easy to implement and which, in the past, has proven effective to make a preliminary study of the core deleptonization during the infall epoch before core bounce, when the collapse is still homologous [4–6]. In Paper I, electron capture was implemented as in the classic work by Bethe et al. (BBAL) [7], but with the strength of capture on nuclei quenched by a factor $\gamma^2 = 0.1$ to account for the Pauli blocking of Gamow–Teller (GT) transitions in neutron-rich heavy nuclei discussed in Refs. [5,8]. The collapse simulation showed that the temperature dependence of the symmetry energy yields a lower rate of neutronization along the collapse, as expected for larger values of E_{sym} , and thence a higher value for the electron fraction, Y_e , at neutrino trapping density. This can be conveniently quantified in terms of the associated *gain* in dissoci-

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ation energy of the shock, $\delta T E_{\text{diss}}$, a quantity which gives a more direct physical insight.¹ The results of Paper I correspond to an energy gain $\delta T E_{\text{diss}} \sim 0.5\text{--}0.6$ foe (1 foe = 10^{51} erg), a non-negligible amount when one considers that the explosion energy (kinetic energy of the ejecta) of SN 1987A was observed to be ~ 1 foe [9].

After Paper I was published, two more investigations of the temperature dependence of the nuclear symmetry energy, due to the presence of neutron-rich heavy nuclei in the collapsing core, have been presented [10,11]; both are based on shell model Monte Carlo (SMMC) calculations, the model of choice to take into account nuclear correlations beyond those treated at the QRPA level. More recently, the virial EOS of hot ($T \gtrsim 2$ MeV) low-density nuclear matter composed of protons, neutrons and alpha particles was obtained [12]; such a scenario is representative of matter around the neutrinosphere after core bounce and shock formation, during the contraction phase of the proto-neutron star and the associated loss of gravitational energy through neutrino emission. The results found in Ref. [12] for the temperature dependence of the symmetry energy follow from the presence of these alpha clusters. Such findings, however, cannot be applied to the pre-bounce scenario studied here; indeed, the temperature of the infalling matter reaches 2 MeV only long after β -equilibrium is achieved and the composition is always dominated by neutron-rich heavy clusters in a sea of unbound neutrons, while the fraction of alphas is negligible at these low temperatures. The T -dependence of E_{sym} , derived in Refs. [2,10,11] and under discussion here, follows from the presence of these exotic nuclear species.

In Ref. [10], several isobaric pairs with mass numbers in the range $A = 54\text{--}64$ were studied. Although the results obtained for the nuclei ^{64}Zn and ^{64}Ni were in agreement with those of Paper I, in their conclusions the authors claimed to “find no systematic temperature dependence of the symmetry energy coefficient,² b_{sym} , for $T \leq 1$ MeV. This contradicts a recent suggestion that b_{sym} increases by 2.5 MeV at this temperature” [10]. An improved SMMC calculation, however, was presented several years later in Ref. [11], where some known problems of the previous paper (small model space and g -extrapolation procedure to circumvent the notorious sign problem of SMMC) had been fixed. Nine isobaric pairs with $A = 56\text{--}66$ were analyzed and this time the authors concluded that their “SMMC studies are consistent with an increase of the symmetry energy with temperature, supporting the argumentation of Donati et al.” [11]. Indeed, upon averaging over the various pairs, they found a variation $\delta b_{\text{sym}} = (6.2 \pm 1.8)\%$ in the temperature interval $T = 0.33\text{--}1.23$ MeV, which is in reasonable agreement with the QRPA results of Paper I, namely an increase of the symmetry energy of $\sim 8\%$ in the interval $T = 0\text{--}1$ MeV.

In the concluding section of Ref. [11], the authors also quickly discussed possible consequences for core-collapse supernovae. They studied the decrease of electron capture on nuclei due to the proposed temperature dependence of the symmetry energy, by considering the increase of reaction Q -values induced by it. For the neutron-rich nuclei expected during collapse (mass number $A > 65$), they adopted new capture rates obtained in the so-called “hybrid” model [13], an approximate approach which mixes SMMC and RPA techniques to go beyond the independent particle model (IPM) in the calculation of both allowed and forbidden transitions. These new results show unblocking of the GT strength [13], due to configuration mixing by the residual interaction and to thermal

excitations, which significantly modify the naive single-particle occupations of the IPM and thus yield capture rates one order of magnitude larger than those predicted by the IPM [5]. Proceeding in this way, the authors obtained changes of electron capture rates due to the T -dependence of b_{sym} that “appear to be rather mild so that one does not expect significant changes for the collapse trajectory” [11]. Although we agree that no dramatic effect on the dynamics of the collapse is to be expected, one should be more cautious in dismissing any significant consequence of the T -dependence of E_{sym} without a collapse simulation. Indeed, not only the reaction Q -values (as considered in Ref. [11]), but also the equation of state of bulk dense matter, the free nucleon abundances, the degree of dissociation into α -particles and the nuclear internal excitations are affected by changes in the symmetry energy. Moreover, the dynamics of the collapse depends in a very non-linear way on the strength of nuclear electron capture,³ so that mild changes in the rates may still result in non-negligible alterations of the overall energetics.

The purpose of the present article is to investigate with a collapse simulation the extent to which the temperature dependence of the nuclear symmetry energy, found in Paper I and confirmed in Ref. [11], can affect the deleptonization of the collapsing stellar core. We must, of course, take into account the remarkable progress made in the SMMC calculations of electron capture rates since publication of Paper I. On the one hand, the new values obtained with improved SMMC techniques for capture on nuclei present in lower-density matter ($A < 65$) [15] have been implemented in modern evolutionary stellar calculations yielding new presupernova models [16], which are significantly different than those used so far as initial conditions in collapse simulations. On the other hand, the unblocked GT strengths found with the hybrid model for the neutron-rich nuclei typical of higher-density matter ($A > 65$) [13] have been used in numerical (1-dimensional) collapse simulations, both Newtonian and relativistic. When compared to the results from the commonly used Bruenn parametrization of nuclear electron capture [17], which quenches capture on heavy nuclei as required by the IPM and thus allows capture on free (unbound) protons to dominate some crucial phases of the collapse, the simulations with the new rates show significant differences in the dynamics of the shock wave and in the neutrino luminosity [14,18,19].

Altogether, the results obtained in the collapse simulations of Paper I have to be revisited in four main aspects, all related to electron capture on nuclei:

- (i) By using the approach of BBAL [7] in Paper I, we certainly overestimated the effect of the temperature-dependent symmetry energy on the deleptonization. Indeed, the BBAL rates for capture on nuclei are calculated applying the Fermi approximation to a shell model description of the GT transition. This statistical limit (which actually does not apply to the collapse scenario, where the shell structure is still dominant and the nuclear density of states is far from thermal [13]) involves an integration over the initial proton states and this multiplies the final capture rates by a factor containing the nucleon effective mass.⁴ This linear dependence of the nuclear rates on m^* obviously amplifies the thermal effects, but it is absent if a more realistic, non-statistical description of capture is adopted.

¹ A larger lepton fraction after trapping corresponds to a larger homologous core so that the shock wave, which forms at its edge after core bounce, will have less material to traverse before getting out of the iron core and thence it will dissipate less energy in the photo-dissociation of tightly bound nuclei [1].

² The coefficient $b_{\text{sym}}(T)$ is the same as the coefficient $s(T)$ of Paper I, and it is related to $E_{\text{sym}}(T)$ by the standard expression $E_{\text{sym}} = b_{\text{sym}}(1 - 2x)^2$, with $x = Z/A$ for a nucleus of mass A and charge Z .

³ The parameter study of Ref. [14], for example, shows that each increase of the rate of capture by a factor 10 corresponds roughly to the same decrease ($\sim 0.1 M_{\odot}$) of the mass of the homologous core.

⁴ The integration requires the nuclear density of states, which in the Fermi gas model is proportional to the nucleon mass [7].

- (ii) The BBAL rates for electron capture (on both nuclei and free protons) used in Paper I were calculated at $T = 0$, but since we are looking for a small thermal effect we cannot neglect the influence of the Fermi distribution functions, which describe the occupation numbers of initial and final particle states at finite temperature [4,5].
- (iii) The multiplying factor $\gamma^2 = 0.1$, introduced in Paper I to account for the Pauli blocking of GT transitions, is not anymore realistic according to the new results from the hybrid model [13]. These new findings, however, are not yet obtained in a consistent SMMC calculation so that, in our opinion, the actual strength of nuclear electron capture is still an open issue and the correct value of γ^2 is not yet pinned down.
- (iv) The initial conditions adopted in Paper I for the collapse have to be revisited, to account for the new results obtained for the presupernova core when implementing the improved SMMC capture rates in evolutionary stellar codes [16].

In the next section, we describe our model for the gravitational collapse of the stellar core and discuss how it takes into proper account all these issues.

2. Physical model for the collapse

In order to study the neutronization of matter induced by gravitational collapse, we develop a one-zone model (sphere of uniform density) along the classic approach of Refs. [4–6]. The model is an improvement over the one used in Paper I in two respects: first, the treatment of electron capture is revisited in order to answer the issues (i) and (ii) previously mentioned; then, the trapping of neutrinos is treated more realistically and provides the equilibrium lepton fraction after trapping, when the collapse is adiabatic. Moreover, the capture strength on nuclei is kept as a free parameter, γ^2 , as discussed in issue (iii), and the presupernova initial conditions are the improved ones mentioned in issue (iv).

We now describe the main features of our collapse model:

- (1) The dynamical evolution of density with time due to gravity decouples from the thermodynamical equations for the changes in entropy and lepton fractions. Therefore, we can follow the relevant thermodynamical variables (entropy, temperature, electron and neutrino fractions, particle abundances, nuclear composition) as a function of density, along the so-called collapse trajectories.
- (2) We adopt the equation of state (EOS) for hot dense matter derived in BBAL [7]. The ensemble of nuclear species is approximated by a mean heavy nucleus⁵ in a sea of dripped-out free neutrons and (fewer) protons. The fractions of free nucleons are determined from nuclear statistical equilibrium. The symmetry energy appears in the bulk nuclear energy and, as a consequence, in the neutron chemical potential, μ_n , and in the neutron–proton energy difference, $\hat{\mu} = \mu_n - \mu_p$. These are crucial quantities in determining the free particle abundances, the nuclear capture Q -values and the entropy changes due the departure from β -equilibrium of the collapsing core before neutrino trapping.
- (3) Thermal dissociation of nuclei into α -particles and nucleons is also taken into account through the Saha equation, but found to have a negligible effect on the collapse trajectories.

- (4) Entropy terms are included for the translational degrees of freedom of all the particles (mean heavy nucleus, free classical nucleons, relativistic degenerate leptons) as well as for the internal nuclear excitations, treated in the Fermi gas approximation. The nuclear excitation energy is proportional to the nucleon effective mass (see Ref. [1]), which is the quantity whose temperature dependence we originally calculated and fitted by an analytical expression in Paper I. We find that the corresponding entropy term has a non-negligible effect on the collapse trajectories.
- (5) Neutrino trapping is set to start at a given trapping density, ρ_{tr} . The typical “standard” value is $\rho_{\text{tr},10} = 43$ (ρ_{10} being the density in units of $10^{10} \text{ g cm}^{-3}$), but we keep it as a model parameter. As long as $\rho < \rho_{\text{tr}}$, neutrinos are allowed to stream freely out of the core and the neutrino fraction is $Y_\nu = 0$. When $\rho \geq \rho_{\text{tr}}$, neutrino diffusion is treated along the lines of Ref. [6]: a degenerate sea of neutrinos with $Y_\nu \neq 0$ is allowed to build up by the inclusion of a diffusion term which decreases with density. Moreover, the inverse reactions induced by the sea of neutrinos are included in the electron capture rates [20], so that weak interactions can reach equilibrium. In this way, complete neutrino trapping is reached gradually at a density somewhat larger than ρ_{tr} ; both the total lepton fraction, $Y_l = Y_e + Y_\nu$, and the entropy tend naturally to constant values, after which the collapse proceeds adiabatically and in β -equilibrium. This is a major improvement over Paper I, where neutrinos were always streaming out freely ($Y_\nu = 0$), so that equilibrium could never be reached and the equilibrium lepton fraction was just the value of the electron fraction taken at $\rho = \rho_{\text{tr}}$ along the collapse trajectory, namely $Y_e = Y_e(\rho_{\text{tr}})$.
- (6) Electron capture is implemented on both free protons and heavy nuclei with standard two-level transitions, as fully developed in Ref. [5]; the phase space integral is calculated numerically, although its approximation by Fermi integral (as in Eq. (1) of Ref. [18]) turns out to be accurate enough. For this kind of transitions, the nuclear capture rate λ_N is a function of density, temperature and two other quantities: the excitation energy of the nuclear GT resonance, Δ_N , and the reaction Q -value. The first is taken as a model parameter, while $Q = \hat{\mu} + \Delta_N$ (we have actually used a regularized expression for the GT excitation energy [20]). We have also multiplied the nuclear strength λ_N by a free parameter, γ^2 . As shown in Ref. [18], the Q -dependence of the capture rates obtained with the hybrid model can be reasonably fitted by the two-level expression, with $\Delta_N = 2.5 \text{ MeV}$ and an appropriate GT matrix element; we normalize λ_N so that our expression coincides with Eq. (1) of Ref. [18] when $\gamma^2 = 1$.
- (7) The temperature dependence of the symmetry energy is treated as in Paper I [2,3], where it was expressed in terms of the T -dependence of the nucleon effective mass, $m^* = m^*(T)$, calculated for different nuclei. The results for each nucleus were fitted with a formula containing two parameters: the value at $T = 0$ of the so-called ω -mass, $m_\omega(0)$, and the temperature scale of this dependence, T_0 . The standard average values are $m_\omega(0) = 1.7$ and $T_0 = 2 \text{ MeV}$, but we keep them as model parameters allowed to vary in a meaningful physical range ($1.4 \lesssim m_\omega(0) \lesssim 1.8$ and $1.9 \lesssim T_0 \lesssim 2.1 \text{ MeV}$ [2]), to account for their dependence on the nucleus studied.

Details and equations can be found in Refs. [2–7]. In particular, we adopted the following differential equations for the collapse trajectories:

- (i) The electron fraction evolution, $dY_e/d\rho$, is Eq. (85) of Fuller [5]. From trapping density onwards, the neutrino-induced inverse

⁵ An ensemble of nuclei is actually present, in nuclear statistical equilibrium under strong and electromagnetic interactions. The mean nucleus is the one that minimizes the nuclear energy and thus it represents the most abundant nuclear species [5].

- reactions are included as in Eqs. (15) and (16) of Ray et al. [6].
- (ii) The entropy and temperature evolution, $dS/d\rho$ and $dT/d\rho$, are respectively: Eqs. (92) and (93) of Fuller before trapping⁶; Eqs. (33) and (35) of Ray et al. after trapping.
- (iii) The neutrino fraction evolution, $dY_\nu/d\rho$, is Eq. (29) of Ray et al. after trapping; before trapping we take $Y_\nu = 0$.

The collapse trajectories are determined starting from a set of initial conditions on the density, ρ_i , the temperature, T_i , and the electron fraction, $Y_{e,i}$ (until trapping density is reached, $Y_\nu = 0$). According to the improved results of Ref. [16] for the central properties of the presupernova core which evolves from a $15M_\odot$ star (about the size of the progenitor of SN 1987A), we will take the initial values $\rho_{10,i} = 0.936$, $T_i = 0.625$ MeV and $Y_{e,i} = 0.432$, which differ significantly from those adopted in Paper I. The differential equations are then integrated and the collapse trajectories of the different thermodynamical quantities are found. In particular, the total lepton fraction $Y_l = Y_l(\rho)$ tends to a constant value, $Y_{l, \text{tr}}$, as the density increases above ρ_{tr} and neutrino trapping is completed.

In the next section, we discuss our results for the neutronization of the core in terms of $Y_{l, \text{tr}}$ and of quantities related to it.

3. Results of the collapse simulation

We first fix the model parameters to their “standard” values ($\rho_{\text{tr},10} = 43$, $m_\omega(0) = 1.7$, $T_0 = 2$ MeV, $\Delta_N = 2.5$ MeV) and make a parameter study of the core neutronization as a function of the nuclear strength in the range $0 \leq \gamma^2 \leq 5$. We point out that $\gamma^2 = 0$ corresponds to electron capture on free protons *only*, while $\gamma^2 = 5$ is very large and probably unrealistic. The older, blocked GT rates of Fuller [5] correspond to $\gamma^2 = 0.1$, while the new unblocked rates of Ref. [18] are associated to $\gamma^2 = 1$. Improved future calculations could change the presently accepted value of the nuclear strength, but (barring discovery of past errors or unexpected breakthroughs) we think that $0.5 \lesssim \gamma^2 \lesssim 2$ should represent a reasonable physical range.

For each choice of parameters, we have run the collapse simulation twice: once implementing the temperature dependence $E_{\text{sym}} = E_{\text{sym}}(T)$ and obtaining $Y_{l, \text{tr}|T}$, once setting $E_{\text{sym}} = E_{\text{sym}}(0)$ and obtaining $Y_{l, \text{tr}|0}$. We indicate by δ_T the “thermal” variation of a quantity *due* to the temperature dependence of the symmetry energy; for example, the thermal change in equilibrium lepton fraction is $\delta_T Y_{l, \text{tr}} = Y_{l, \text{tr}|T} - Y_{l, \text{tr}|0}$.

From the collapse trajectories of the lepton fraction, $Y_l = Y_l(\rho)$, we can infer the density range where the temperature dependence of the symmetry energy is most relevant. Regarding the thermal variation of the capture rates, we obtain similar results as those shown in Fig. 7 of Ref. [11]: the thermal effect is maximum at the onset of collapse ($\rho_{10} \sim 1$) and decreases with increasing density; when trapping sets in ($\rho_{10} \sim 40$), the difference in the capture rates due to $E_{\text{sym}}(T)$ is already negligible. However, since this is a cumulative and non-linear effect on the lepton fraction, a difference between the collapse trajectories, $\delta_T Y_l(\rho) = Y_l(\rho)|_T - Y_l(\rho)|_0$, gradually builds up as the density increases. This is shown in Fig. 1, where the thermal variation of the lepton fraction collapse trajectories, $\delta_T Y_l$, is given as a function of the density of the collapsing core. We notice that, even though the thermal variation actually reaches its final equilibrium value, $\delta_T Y_{l, \text{tr}}$, only when neutrino trapping is fully achieved (around $\rho_{10} \sim 250$), the whole effect is seen to build up before trapping sets in. In particular, it is mostly in the low-density range $1 \lesssim \rho_{10} \lesssim 20$ that the tempera-

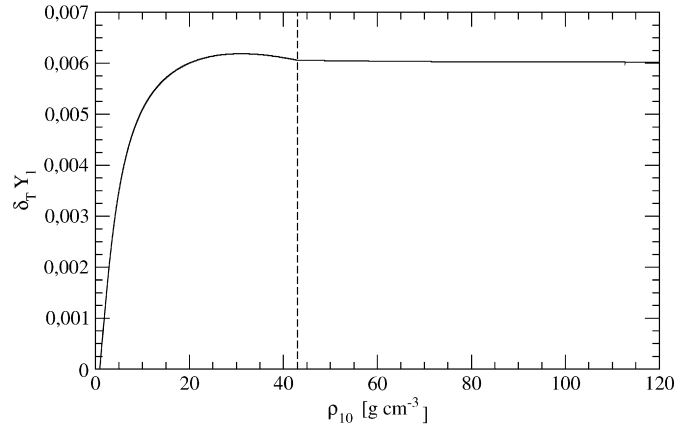


Fig. 1. Thermal variation of the lepton fraction collapse trajectories, $\delta_T Y_l$, as a function of the density of the collapsing core, ρ_{10} (in units of $10^{10} \text{ g cm}^{-3}$). The dotted line indicates the trapping density, ρ_{tr} . The curve corresponds to standard parameters of the model ($\rho_{\text{tr},10} = 43$, $m_\omega(0) = 1.7$, $T_0 = 2$ MeV, $\Delta_N = 2.5$ MeV).

Table 1

Results of the collapse simulation for different values of the strength of nuclear electron capture, γ^2 . We show the equilibrium lepton fractions after trapping obtained without, $Y_{l, \text{tr}|0}$, and with, $Y_{l, \text{tr}|T}$, the temperature dependence of the symmetry energy; the thermal change of the equilibrium lepton fraction, $\delta_T Y_{l, \text{tr}}$; the corresponding gain in dissociation energy of the shock, $\delta_T E_{\text{diss}}$ (in foe). The model parameters are the standard ones ($\rho_{\text{tr},10} = 43$, $m_\omega(0) = 1.7$, $T_0 = 2$ MeV, $\Delta_N = 2.5$ MeV).

γ^2	$Y_{l, \text{tr} 0}$	$Y_{l, \text{tr} T}$	$\delta_T Y_{l, \text{tr}}$	$\delta_T E_{\text{diss}}$ (foe)
0	0.3996	0.4054	0.0058	0.45
0.1	0.3802	0.3861	0.0060	0.44
0.5	0.3460	0.3519	0.0059	0.41
1	0.3291	0.3351	0.0060	0.39
2	0.3114	0.3175	0.0061	0.38
5	0.2808	0.2868	0.0060	0.34

ture dependence of the symmetry energy affects significantly the neutronization process.

Surprisingly, although Fig. 1 corresponds to standard parameters of the model, the curve $\delta_T Y_l(\rho)$ is found to change very little under physically reasonable variations of the model parameters, as will be discussed later on.

The results for the equilibrium lepton fractions and the associated thermal changes are shown in Table 1 for different values of γ^2 . We point out how the general magnitude of the equilibrium lepton fraction is a very slowly decreasing function of γ^2 . Increasing the strength by a factor ten, from the blocked to the unblocked capture rates, decreases the equilibrium lepton fraction by $\sim 13\%$, which is in reasonable agreement with the $\sim 10\%$ change obtained in Newtonian one-dimensional simulations [14]. We also notice that the thermal effect under study *systematically* reduces the equilibrium neutronization, namely $Y_{l, \text{tr}}$ is increased by an almost constant value, $\delta_T Y_{l, \text{tr}} \simeq 0.006$, irrespective of the value of the strength γ^2 . Although small, this effect is not negligible, as we will argue in the remaining of this article.

In order to determine the relevance of our results to supernova explosions, we need a quantity with a more direct physical meaning and which can be compared to relevant observables. As in Paper I, we use the gain in shock dissociation energy which is defined as $\delta_T E_{\text{diss}} = 98[(Y_{l, \text{tr}|T})^2 - (Y_{l, \text{tr}|0})^2] = 98 \delta_T Y_{l, \text{tr}}^2$ (in foe). Although based on a schematic model for the shock formation and propagation [21], this expression provides a reasonable order of magnitude estimate of $\delta_T E_{\text{diss}}$. In a similar fashion, one could consider the change in initial (i.e. post-bounce) shock energy, $\delta_T E_{\text{shock}}$, which also follows from changes in the equilibrium lepton fractions affecting the size of the homologous core.

⁶ In Eq. (92) of Fuller, the terms AX_p in the denominators should each be multiplied by a factor λ_{fp} .

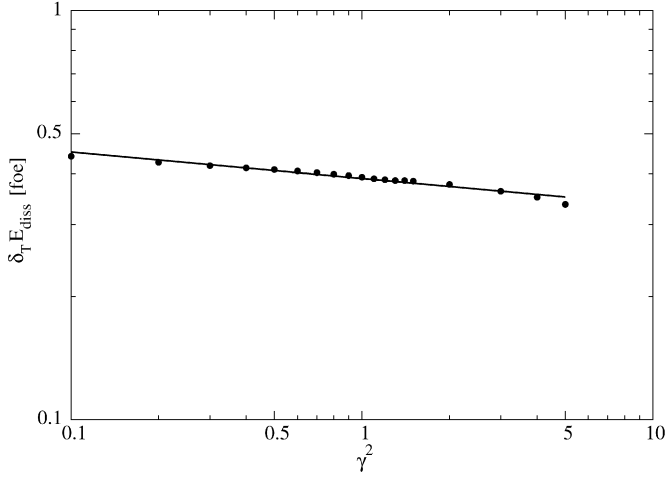


Fig. 2. Gain in dissociation energy of the shock, $\delta_T E_{\text{diss}}$ (in foe), as a function of the strength of nuclear electron capture, γ^2 . The calculated points correspond to standard parameters of the model ($\rho_{\text{tr},10} = 43$, $m_\omega(0) = 1.7$, $T_0 = 2$ MeV, $\Delta_N = 2.5$ MeV). The line represents a power-law fit, with exponent $m = -0.065$.

Table 2

Dependence of the results from the parameters of the model. We show the gain in dissociation energy of the shock, $\delta_T E_{\text{diss}}$ (in foe), for different values of the strength of nuclear electron capture, γ^2 . In each column we change only the value of one parameter, either the excitation energy of the GT resonance, Δ_N (in MeV), or the trapping density, $\rho_{\text{tr},10}$, while the other parameters are the standard ones ($\rho_{\text{tr},10} = 43$, $m_\omega(0) = 1.7$, $T_0 = 2$ MeV, $\Delta_N = 2.5$ MeV).

γ^2	$\Delta_N = 2$	$\Delta_N = 3$	$\Delta_N = 4$	$\rho_{\text{tr},10} = 35$	$\rho_{\text{tr},10} = 55$
0	0.45	0.45	0.45	0.43	0.46
0.1	0.43	0.45	0.46	0.44	0.44
0.5	0.40	0.42	0.43	0.42	0.38
1	0.38	0.40	0.42	0.42	0.36
2	0.37	0.38	0.40	0.40	0.34
5	0.28	0.35	0.38	0.37	0.28

In the schematic approach of Ref. [5], however, the expression for the initial shock energy, $E_{\text{shock}} = E_{\text{shock}}(Y_{l,\text{tr}})$, has a maximum for $Y_{l,\text{tr}} = \frac{10}{13} Y_i = 0.3323$. Since the equilibrium lepton fractions corresponding to $\gamma^2 = 1$ are close to this extremum (cf. Table 1), the thermal effect $\delta_T E_{\text{shock}}$ turns out to be quite small ($\sim 10^{-2}$ foe); we will not consider it in the following.

Since $\delta_T Y_{l,\text{tr}}$ is small, the thermal gain in dissociation energy can be written as $\delta_T E_{\text{diss}} \simeq 196 Y_{l,\text{tr}|0} \times \delta_T Y_{l,\text{tr}}$. This shows that in general $\delta_T E_{\text{diss}}$ depends on $\delta_T Y_{l,\text{tr}}$, but its magnitude is fixed by the final neutronization reached by matter, $Y_{l,\text{tr}|0}$, which is determined by the nuclear capture strength γ^2 . In the last column of Table 1, we show the results for the gain in dissociation energy. For standard parameters and $\gamma^2 = 1$, we find $\delta_T E_{\text{diss}} = 0.39$ foe. Moreover, since $\delta_T Y_{l,\text{tr}}$ is constant, the gain in dissociation energy has the same very slow dependence on the strength parameter as the equilibrium lepton fraction. This is well seen in Fig. 2, where $\delta_T E_{\text{diss}}$ is given as a function of γ^2 . The points are the results of the collapse simulation, while the line in the log-log graph represent a power-law best fit, with a very small exponent $m = -0.065$. In the physical meaningful range for the strength ($0.5 \lesssim \gamma^2 \lesssim 2$), the gain in dissociation energy varies only by $\pm 4\%$, in the interval $\delta_T E_{\text{diss}} \sim 0.38\text{--}0.41$ foe.

Although the previous discussion indicate a quite stable value $\delta_T E_{\text{diss}} \sim 0.4$ foe, we want to study the robustness of such a result under reasonable variations of the model parameters, compatible with present theoretical uncertainties about the values of $\rho_{\text{tr},10}$, Δ_N , $m_\omega(0)$ and T_0 . In Table 2, we show $\delta_T E_{\text{diss}}$ for different values of γ^2 : each column represents the case in which only one of the parameters, $\rho_{\text{tr},10}$ or Δ_N , is changed from its standard value to

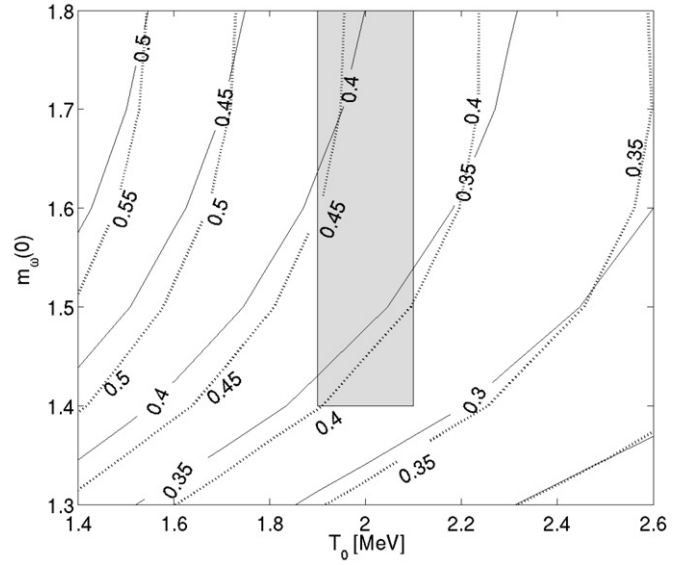


Fig. 3. Gain in dissociation energy of the shock, $\delta_T E_{\text{diss}}$ (level lines labelled in foe), as a function of the parameters $m_\omega(0)$ and T_0 . The solid contour lines correspond to $\gamma^2 = 1$, the dotted contour lines to $\gamma^2 = 0.1$. The other parameters are the standard ones ($\rho_{\text{tr},10} = 43$, $\Delta_N = 2.5$ MeV). The shaded area shows the physical range found in Ref. [2] for the parameters $m_\omega(0)$ and T_0 .

the value indicated. In Fig. 3, instead, we show a contour plot for $\delta_T E_{\text{diss}}$ (in foe) as a function of the two parameters $m_\omega(0)$ and T_0 , the other ones being fixed at their standard values. The solid level lines are for $\gamma^2 = 1$, the dotted ones for $\gamma^2 = 0.1$ and the shaded area indicates the physically meaningful range found in Paper I for the thermal parameters of the symmetry energy.

The results of Table 2 and Fig. 3 show that, under reasonable variations of the model parameters, the gain in dissociation energy of the shock changes only by about $\pm 10\%$, in the range $\delta_T E_{\text{diss}} \sim 0.35\text{--}0.45$ foe. This proves the robustness of our conclusions: the temperature dependence of the symmetry energy yields a systematic energy gain (less dissipation of shock energy), whose order of magnitude is $\delta_T E_{\text{diss}} \sim 0.4$ foe.⁷ In the concluding section, we will discuss the relevance of such a result for the physics of supernova explosions.

4. Conclusion

In this Letter we have studied the effect of the temperature dependence of the symmetry energy on the neutronization processes occurring during gravitational collapse in a supernova explosion. We have assumed for E_{sym} the T -dependence found in Paper I and later confirmed in Ref. [11], first fixing the parameters to their average values $m_\omega(0) = 1.7$, $T_0 = 2$ MeV, but later allowing them to vary in a reasonable physical interval. We have followed the collapse with a one-zone model, finding the collapse trajectories of the different thermodynamical variables (temperature, entropy, particle abundances, lepton fractions, mean heavy nucleus) and determining the equilibrium lepton fraction after neutrino trapping, when the collapse becomes adiabatic. We have implemented electron capture on both free protons and nuclei with standard two-level transitions at finite temperature. However, to account for the present theoretical uncertainties concerning electron capture rates in exotic nuclei, we have multiplied the nuclear strength re-

⁷ As in Paper I, we have set the volume and symmetry energy coefficients in the BBAL EOS to the values $w_0 = -16.5$ MeV and $s(0) = 29.3$ MeV, respectively. Different values give a different energy gain, but do not alter our general conclusions (for example, with $w_0 = -16$ MeV and $s(0) = 31.3$ MeV we find $\delta_T E_{\text{diss}} \sim 0.3$ foe).

cently obtained in Ref. [18] by a strength parameter γ^2 : variations in a range $0.5 \lesssim \gamma^2 \lesssim 2$ around the presently accepted value of $\gamma^2 = 1$ are not to be ruled out in the future. Starting from the improved presupernova initial conditions of Ref. [16], we have run the collapse simulation with and without the T -dependence of E_{sym} implemented, thus obtaining the “thermal” change in deleptonization, $\delta_T Y_{l,\text{tr}}$. Then, we have studied the significance of this thermal effect in terms of a quantity with more direct physical meaning, the corresponding gain in dissociation energy of the shock, $\delta_T E_{\text{diss}} \propto \delta_T Y_{l,\text{tr}}^2$. Finally, we have tested the solidity of our results by varying the standard parameters of the model ($\rho_{\text{tr},10} = 43$, $m_\omega(0) = 1.7$, $T_0 = 2$ MeV, $\Delta_N = 2.5$ MeV) within reasonable physical ranges, compatible with present theoretical uncertainties.

The main conclusion of our investigation is that the temperature dependence of the symmetry energy systematically reduces the neutronization of the core, namely it consistently increases the equilibrium lepton fraction by a small constant amount, $\delta_T Y_{l,\text{tr}} \simeq 0.006$, irrespective of the value of the strength parameter γ^2 . The corresponding gain in shock dissociation energy, instead, decreases with increasing nuclear strength, but very slowly: when $\gamma^2 = 1$ is divided or multiplied by two, $\delta_T E_{\text{diss}}$ varies only by $\pm 4\%$ around its standard parameter value $\delta_T E_{\text{diss}}|_{\gamma^2=1} = 0.39$ foe. Moreover, significant changes in the other one-zone model parameters correspond to a quite small range $\delta_T E_{\text{diss}} \sim 0.35\text{--}0.45$ foe. This confirms the robustness of our results and the presence of a systematic gain in shock dissociation energy of order $\delta_T E_{\text{diss}} \sim 0.4$ foe, associated to the temperature dependence of the symmetry energy.

Such an effect is obviously not a dramatic one, when one considers that the total energy sapped from the shock by photodissociation of nuclei is larger by almost two orders of magnitude. Indeed, even changing the nuclear strength by a factor of ten through the unblocking of GT transitions does not qualitatively alter the final outcome of the failed explosion, at least in one-dimensional simulations [19]. Actually, recent developments of three-dimensional simulations of core-collapse supernovae indicate that the roles of neutrinos, fluid instabilities, rotation and magnetic fields are probably critical to obtain successful explosions [22]. However, when compared to the typical kinetic energies of a supernova explosion, K_{expl} , which are imparted by the shock wave to the ejecta, a gain in shock energy of $\delta_T E_{\text{diss}} \sim 0.4$ foe is not negligible (for SN 1987A, observation gave $K_{\text{expl}} \sim 1$ foe [9]). Moreover, $\delta_T E_{\text{diss}}$ is two orders of magnitude larger than the total electromagnetic output [1]. On general grounds, since both the explosion energy K_{expl} and the much smaller electromagnetic output have small values resulting from differences of very large quantities (gravitational energy, initial post-bounce shock energy, neutrino losses, nuclear photo-dissociation), it follows that the explosion observables can be sensitive to subtle microphysical features. In particular, systematic nuclear effects can be of particular importance, as noted also in the conclusions of Ref. [14].

The numerical results of our one-zone collapse simulation are significant for their order of magnitude, not their precise values which are limited by the oversimplified zero-dimensional ap-

proach. In our opinion, their robustness under variations of the model parameters justifies further investigation in detailed one-dimensional numerical codes. This is not a straightforward task, since not only the reaction Q -values, but the whole EOS describing dense hot matter is affected by the T -dependence of E_{sym} . In particular, this is lengthy to implement within the Lattimer and Swesty EOS [23], currently used in realistic supernova codes, since changing any of its nuclear input parameters (in the present case introducing $m_\omega(0)$ and T_0 to parameterize $m^* = m^*(T)$ and thence the temperature dependence of the symmetry energy) requires recalculating the phase and Maxwell construction boundaries before generating the final table of the EOS. We are presently working along these lines [24], with interesting preliminary results from one-dimensional simulations with the simpler BBAL EOS, where the temperature dependence of the symmetry energy can be implemented analytically.

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