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Realistic energies for vortex pinning in intermediate-density neutron star matter

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Abstract

Realistic values for the pinning energies of vortices in the neutron superfluid expected in the inner crust of neutron stars are crucial for the theory of pulsar glitches. To this end, we supplement our consistent semi-classical model for the vortex–nucleus interaction with general properties of intermediate-density fermion systems with large negative scattering lengths, such as neutron matter at the densities corresponding to the inner crust. We also implement the reduction of pairing expected from the polarization of the strongly correlated neutron medium, although allowing for the present large theoretical uncertainties on the amount of reduction. Finally, we better evaluate the kinetic contributions to pinning accounting also for the quantum structure of the vortex core, which sustains divergenceless flow. When compared to existing results, we find *weaker* values for the pinning energies per site ($E_P < 3.5$ MeV); moreover, significant nuclear pinning occurs only in a *restricted* density range (about $2 \times 10^{13} \lesssim \rho \lesssim 5 \times 10^{13}$ g/cm^3 or $0.07\rho_0 \lesssim \rho \lesssim 0.2\rho_0$, with ρ_0 the nuclear saturation density). The rest of the crust presents either interstitial pinning ($\rho < 0.07\rho_0$) or collective super-weak pinning ($\rho > 0.2\rho_0$), both negligible at the macroscopic scale relevant to vortex unpinning and glitches.
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1. Introduction

Pulsar glitches, namely the sudden spin-ups in the otherwise decreasing frequencies of rotating neutron stars [1], may be the most compelling observational signature of the existence of macroscopic superfluidity inside these dense astrophysical objects, possibly the only systems in the Universe where such a property can naturally be found in bulk matter. Indeed, according to the vortex model [2], pulsar glitches are due to the sudden transfer of angular momentum from the array of vortices threading the neutron superfluid to the normal component of the star, which is coupled to and co-rotates with the electromagnetic field. The pinning sites required by this scenario can be found in the inner crust of the star, namely in the density range $\rho_d \lesssim \rho \lesssim 0.6\rho_0$ [3] where $\rho_d = 4 \times 10^{11}$ g/cm^3 is the neutron drip density and $\rho_0 = 2.8 \times 10^{14}$ g/cm^3 is the nuclear saturation density. Here, the predicted co-existence [4] of a Coulomb lattice of heavy and very neutron-rich nuclei (the nuclear impurities) together with a background of unbound superfluid neutrons naturally provides the sites to which vortices can bind.

Realistic values for the pinning energies and forces are obviously crucial to test the validity of the vortex model against actual observations of glitches. The first studies [5,6], based on different approximations, predicted an extended density range ($0.01\rho_0 \lesssim \rho \lesssim 0.5\rho_0$) with quite strong pinning, too strong to agree with observational data. Indeed, the critical difference of angular velocity for vortex unpinning, $\Delta\Omega_{\rm cr}$, can be constrained (although not in a model-independent way) by ob-

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servations of X-ray emission from pulsars of different age. Several studies of the thermal evolution of superfluid pulsars have provided an upper bound for the average critical angular velocity, namely they find that $\overline{\Delta\Omega_{\rm cr}} \lesssim 10$ rad/s in order to be consistent with the observed luminosity of older pulsars [7]. Order of magnitude estimates of the critical velocities using the early pinning energy calculations, instead, resulted in $\Delta\Omega_{\rm cr} \sim 10$–$20$ rad/s [8] and $\Delta\Omega_{\rm cr} \sim 10$–$100$ rad/s [9], both in obvious disagreement with present observational data.

In a series of papers [10–12] we have developed a consistent semi-classical model for the vortex-nucleus interaction in the inner crust of neutron stars. Our approach is based on applying the Thomas–Fermi ansatz in the Local Density Approximation (LDA) to describe the pairing properties of the system, consistently requiring thermodynamical and mechanical equilibrium throughout the system. Physical details about the model are given in Ref. [12] (from now on Paper I), as well as comments on the validity of such an approach.

As compared to the previous estimates, the fully consistent semi-classical model presented in Paper I predicted a more limited density range ($0.07\rho_0 \lesssim \rho \lesssim 0.5\rho_0$) of significantly weaker pinning, with average pinning forces one order of magnitude smaller than those obtained with the Landau–Ginzburg approximation [6,9]. Waiting for a consistent and credible quantum calculation, to our knowledge the semi-classical LDA model is to date the most predictive one, as it generally yields the right orders of magnitude of the volume-integrated energies although often over-estimating their numerical values (cf. Ref. [13] and the discussion in Paper I).

In order to improve the LDA estimates and get more stringent limits on the pinning parameters, which is the goal of the present Letter, we must supplement the model with additional physical information. Correcting the model like this may spoil its full consistency, but altogether it makes it more realistic by overcoming some unphysical previous assumptions. One can try to improve the treatment of all the terms which contribute to the pinning energy (Fermi internal, pairing condensation, kinetic, nuclear). Here we intervene on the first three terms while the last one, connected to the nuclear composition of the inner crust, will be discussed in a forthcoming paper.

Recent studies about the properties of systems of Fermions interacting via short-range attractive forces with large negative scattering lengths have derived some important general results which are valid in the so-called *intermediate*-density regime [14–16]. The unbound neutrons in the inner crust of neutron stars are to a reasonable approximation in such a regime, particularly so in the external layers of the crust. Here we implement the general energy behavior expected at intermediate-densities by rescaling the Fermi internal terms. Physically, this corresponds to (roughly) introducing into the model the mutual attractive interaction between neutrons. Such a contribution is missing in the LDA approach, where the neutrons are locally described as a degenerate gas of non-interacting particles so that their Fermi internal energy is that of a free system. Basically, this correction restores the correct energy scale of the interacting system, lost in the LDA model.

In our previous studies we have used two different pairing gaps to describe the condensation energy due to superfluidity, namely those obtained in the BCS approximation with a realistic (Argonne) and an effective (Gogny) interaction. It is well known, however, that the polarization of the strongly correlated neutron medium gives both self-energy and vertex corrections [17], which altogether amount to a strong suppression of the pairing gap. Here we introduce this effect by artificially rescaling the Argonne pairing gap. Indeed, although the different calculations beyond BCS do not yet agree on the amount of the suppression [18], most of them correspond to a reduction by a factor between 2 and 3 of the BCS gap obtained with realistic interactions. This range of uncertainty is a reflection of our present poor understanding of pairing in dense Fermi systems and to date it cannot be avoided.

When treating the kinetic term due to the vortex flow, a cutoff was introduced in the LDA model in order to preserve the hydrodynamic continuity equation for the superfluid. Deeper in the crust, however, where the pinning gets weaker, the neglected energy contribution can become significant. Here we estimate this missing kinetic contribution, due to the presence of the nucleus in the vortex flow, using the approximate analytical results of Ref. [6] which, although not fully consistent, are still compatible with the model.

Still concerning the kinetic contributions, in the LDA the vortex core is made of normal matter at rest. Quantum calculations of vortices, however, have shown that the core sustains a macroscopic flow which vanishes only at the vortex axis [19–21]. Here we estimate this additional kinetic contribution due to the core on the basis of the general quantum results of Ref. [19]. Altogether, the two kinetic corrections are calculated in the same approximation (uniform densities of the nucleus and the neutron gas [11]) and they turn out to be of comparable magnitude and not negligible in the middle of the inner crust.

Applying these four corrections to the semi-classical model gives more stringent limits on the pinning properties of the crust. Now we find *weaker* values for the pinning energies per site ($E_P < 3.5$ MeV); moreover, nuclear pinning, the only one significant at the macroscopic scale, occurs only in a *restricted* density range (about $2 \times 10^{13} \lesssim \rho \lesssim 5 \times 10^{13}$ g/cm^3 or $0.07\rho_0 \lesssim \rho \lesssim 0.2\rho_0$). The rest of the crust presents either interstitial pinning (for $\rho < 0.07\rho_0$) or collective super-weak pinning (for $\rho > 0.2\rho_0$), both situations corresponding to negligible pinning energy per unit length, which is the macroscopic energy necessary to effectively unpin a vortex and therefore is the one relevant to pulsar glitches [8,9].

In Section 2 we present the modifications introduced in the LDA model, in Section 3 we show the results of the calculation and Section 4 contains some conclusive remarks.

2. Physical scenario

The LDA model for the vortex-nucleus interaction has been fully presented in Paper I. We take that paper as our starting point, keeping the same notation and only discussing modifications to the approach. We remind that we are studying five zones along the inner crust, whose microscopic properties where cal-

Table 1
Physical parameters of the five zones of interest as calculated in Ref. [4]. The average baryon densities, ρ_B, are given in $\mathrm{g/cm}^3$, the densities of the free neutron gas, n_G, in fm^{-3}, the inverse Fermi momenta, $k_{F,G}^{-1}$, the radii of the WS-cells, R_{WS}, and those of the nuclei, R_N, in fm

Zone	1	2	3	4	5
ρ_B	1.5×10^{12}	9.6×10^{12}	3.4×10^{13}	7.8×10^{13}	1.3×10^{14}
n_G	4.8×10^{-4}	4.7×10^{-3}	1.8×10^{-2}	4.4×10^{-2}	7.4×10^{-2}
$k_{F,G}^{-1}$	4.2	1.9	1.2	0.9	0.8
R_{WS}	44.0	35.5	27.0	19.4	13.77
R_N	6.0	6.7	7.3	6.7	5.2

culated in Ref. [4]. In Table 1 we summarize some quantities of interest here: the mean baryon density, ρ_B; the unperturbed density of the unbound neutron gas away from the nuclear impurity, n_G; the corresponding Fermi momentum, $k_{F,G}$ with $k_{F,G} = (3\pi^2 n_G)^{1/3}$; the radius of the Wigner–Seitz unit cell, R_{WS}, and that of the central neutron-rich nucleus, R_N.

As a general comment about the density range studied here, exotic nuclear phases (the so-called ‘nuclear pasta’ regime [3]) should appear for densities $0.4\rho_0 \lesssim \rho \lesssim 0.6\rho_0$, so that zone 5 would actually consist of cylindrical rodlike nuclei rather than spherical ones. The pinning problem becomes quite involved with these exotic phases, and one could even imagine strong pinning in very particular but unlikely configurations (e.g. if the vortices are co-axial with the ‘spaghetti’ nuclei, which could happen only if the lattice of rod nuclei is perfectly aligned with the pulsar rotational axis). On general grounds, however, the pinning phenomenon is originated by the difference in pairing energy density existing between the dense matter in the nucleus and the more dilute external neutron gas. Since this difference decreases as the density approaches ρ_0, it is reasonable to expect smaller pinning energies as one moves into the deeper regions of the crust where exotic phases exist: zones 3, 4 and 5 already show this decrease of pinning with depth for spherical nuclei (cf. Table 6).

We start by discussing the internal energy. In Paper I, apart from the Woods–Saxon potential representing the central nucleus, it was taken as the purely kinetic contribution of a gas of non-interacting completely degenerate neutrons (Fermi model). This amounts to neglecting the negative contribution from the attractive nucleon–nucleon interaction, which would reduce the total internal energy. The density range in the inner crust, however, allows us to somehow restore the correct scale of energies. Indeed, the S-wave scattering length for neutrons ($a_{nn} \sim -18.8$ fm) is much longer than the typical range of the strong interaction potential ($r_0 \sim 1$ fm). Degenerate uniform Fermi systems interacting via short-range attractive forces with long negative scattering lengths ($r_0 \ll |a|$) have been recently studied with different approaches, all reaching similar conclusions [14–16].

The value of the inter-particle spacing, $\sim k_F^{-1}$ with $k_F = (3\pi^2 n)^{1/3}$, defines different density regimes. In the *intermediate*-density region, $r_0 \lesssim k_F^{-1} \lesssim |a|$, because of the absence of length scales other than k_F^{-1} in the limit $r_0 \ll |a|$, the energy per particle is found to scale with the Fermi energy, namely $E/N = \alpha \frac{3}{5} E_F$, while it does not depend on the scattering length, range or other details of the interaction [15,16]. These general results are not just a finite-order perturbative expansion around the non-interacting gas, but rather they represent a full result for the interacting theory in the limit $r_0 \ll |a|$.

Table 1 shows that the inner crust is approximately in the intermediate-density regime, particularly so in its outer layers (zones 1–3). Indeed, $k_{F,G}^{-1} < |a_{nn}|$ holds always, while the condition $k_F^{-1} \gtrsim r_0$ holds in the first three zones whereas it marginally breaks down in the other two zones. The general results mentioned before were obtained for a uniform system, but in the spirit of the LDA we will apply them locally. Therefore, we substitute Eq. (1) of Paper I for the Fermi internal terms with the following definitions

$$\varepsilon_\alpha(\mathbf{x}) = \alpha \varepsilon_{\mathrm{fer}}(\mathbf{x}) = \alpha \frac{3}{5} \frac{\hbar^2 k_f^2(\mathbf{x})}{2m_n} n(\mathbf{x}),$$
$$\mu_\alpha(\mathbf{x}) = \alpha \mu_{\mathrm{fer}}(\mathbf{x}) = \alpha \frac{\hbar^2 k_f^2(\mathbf{x})}{2m_n}, \tag{1}$$

where m_n is the neutron mass. In this way, we are able to (roughly) compensate for the absence of the attractive potential term in the internal energy. The parameter α has also been calculated and within $\sim 10\%$ the results are in reasonable agreement [14–16]. We will take the typical value $\alpha = 0.5$ for neutron matter at intermediate densities; moreover, we have checked that, under $\pm 10\%$ variations of the parameter α, our final general conclusions about the crust pinning properties still hold.

We next discuss the condensation energy term. In Paper I we considered two density-dependent pairing gaps, $\Delta = \Delta(n)$, both obtained in the BCS (mean field) approximation, one with a realistic microscopic nucleon–nucleon interaction (Argonne), the other with an effective potential (Gogny). It is well known, however, that polarization effects of the neutron medium will strongly reduce the pairing gap [17]. The amount of such a suppression is still an open issue in the dense-matter scientific community, since going consistently beyond the mean field approximation has been so far only partially achieved. Looking at the results from different calculations, however, we see that most of them roughly correspond to a reduction of the BCS gap calculated with a realistic interaction by a factor between two and three [18]. Therefore, in order to account for medium correlation effects on pairing, we take the Argonne gap used in Paper I and rescale it as

$$\Delta(n)\big|_\beta = \frac{1}{\beta} \Delta(n)\big|_{\mathrm{Argonne}}, \tag{2}$$

where the reduction factor β can vary between 2 and 3. The interval of uncertainty $2 \lesssim \beta \lesssim 3$ represents the extent of our present ignorance, and for the time being it is unavoidable.

We finally discuss the kinetic energy term. As already mentioned, we add two corrections to what obtained in Paper I: the first one is related to the presence of the nucleus in the vortex flow, the other one to the quantum structure of the vortex core. In Paper I, when integrating over the Wigner–Seitz cell the kinetic term arising from the superfluid vortex flow, $\varepsilon_{\mathrm{kin}}(\mathbf{x}) = \frac{\hbar^2}{8m_n R^2} n(\mathbf{x})$ where R is the distance from the vortex axis, we introduced a cutoff on R in order not to violate the continuity equation and thus preserve hydrodynamical consistency. This cutoff amounts to neglecting the change in kinetic energy with respect to the unperturbed vortex flow, due to the presence of the nucleus at some distance away from the vortex axis. We can still estimate this contribution using the analytical results obtained by the authors of Ref. [6] for a schematic model, consisting of a uniform gas of unbound neutrons with density n_G and of a spherical nucleus of radius R_N and constant neutron density n_N. They calculate the superfluid flow when the center of the nucleus is at a distance D from the vortex axis, enforcing the continuity equation and the potential nature of the velocity field, and they obtain for the change in kinetic energy from the presence of the nucleus the expression

$$\Delta K_N \approx -\frac{3}{2} M_d \frac{\lambda - 1}{\lambda + 2} \left(\frac{\hbar}{2m_n}\right)^2 \frac{1}{D^2}, \tag{3}$$

where $\lambda = n_N / n_G$ and $M_d = \frac{4}{3}\pi R_N^3 m_n n_G$. We introduced a minus sign to be consistent with the definitions of ΔE given in the next section. For the neutron density inside the nucleus we will take the results of Ref. [4], namely $n_N \approx 0.1\ \mathrm{fm}^{-3}$. Although just an approximate result, Eq. (3) should give at least the right order of magnitude for the kinetic correction to the LDA model, ΔK_N, due to the presence of the nucleus in the vortex flow.

We next turn to the quantum structure of the vortex core and its kinetic contribution to pinning. We proceed in three steps:

(i) We first observe that the kinetic term $\varepsilon_{\mathrm{kin}}$ used in Paper I corresponds to the axially symmetric velocity flow $v(R) \propto 1/R$, which is obtained for a vortex in classical hydrodynamics. This hydrodynamical approximation obviously breaks down as one approaches the vortex axis, where the hydrodynamical flow diverges. On the other hand, it represents quite well the superfluid flow *outside* the vortex core, where the length scale associated to the flow, $|\frac{d \ln v}{dR}|^{-1} = R$, is larger than both the mean free path of the strongly interacting neutrons and the pairing coherence length $\xi \sim \frac{\hbar^2 k_F}{m_n \Delta}$, the characteristic microscopic scale of the neutron superfluid which represents the 'size' of the Cooper pairs of fermions. Indeed, together these two conditions justify the LDA description of the superfluid kinematics outside the vortex core in terms of classical irrotational flow.

(ii) We next turn to the core structure. In the LDA we assumed that the vortex core is made of normal matter at rest and therefore without any kinetic contribution. Quantum calculations of vortices, however, have shown that the core sustains a macroscopic flow which, as opposed to the hydrodynamical flow, does not diverge as $R \to 0$. Several studies for systems of bosons, from the Gross–Pitaevsky equations in the Hartree approximation [22] to Fetter's remarkable paper on the core structure of a quantized vortex [19], yield a linear flow profile along the vortex core. This flow is associated to the large component of 'non-condensed' particles of the interacting system, whose density is roughly constant throughout the fluid, including the vortex core. Later studies for systems of fermions undergoing pairing based on different methods, from the Bogoliubov–de Gennes theory [20] to the energy density functional approach [21], show a similar behavior, but with the pairing coherence length ξ in place of the bosonic characteristic length and with the mass of a Cooper pair in place of the boson mass. As expected, all these studies naturally yield the hydrodynamical flow away from the core. The main general conclusion about the quantum structure of the vortex core is that it can sustain divergenceless superfluid rotations with roughly constant particle density throughout the fluid, including the vortex core region and its axis. Therefore, we can correct the LDA model by allowing the normal core to rotate as predicted by quantum theory; we will use approximate analytical expressions for the flow, so that we expect our approach to provide only order of magnitude estimates of such a correction. Following the arguments of Ref. [19], we then assume a constant non-zero vorticity *inside* the vortex core (cf. Eq. (6) of Ref. [19]), namely $|\vec{\nabla} \times \vec{v}| \approx \hbar/(m_n \xi^2)$ for $R < \xi$, corresponding to a 'rigid body' rotation $v(R) = \omega R$ with angular velocity $\omega = \hbar/(2m_n \xi^2)$. This flow merges continuously into the hydrodynamical flow at $R = \xi$. Note that, following Fetter's arguments, we have assumed for $R \to 0$ a finite (non-zero) matter density comparable to the density at $R \to \infty$. This is consistent with our LDA description of the core in the 'mixed' phase, where no significant matter depletion occurs in the core of normal matter, as opposed to the hollow core of the 'pure phase' (cf. next section). Also the velocity profile is consistent with the LDA normal core, since normal matter can sustain rigid rotations, as opposed to irrotational superfluid matter.

(iii) Finally, we must determine the kinetic contribution to pinning from the matter flow predicted inside the vortex core by quantum theory. A moment of reflection shows that, within the LDA approach, the *difference* ΔK_ξ between the contribution of the vortex core in the NP and IP configurations (cf. next section) is only due to a region corresponding to the volume occupied by the nucleus, since in LDA there is exact cancellation between the regions away from the nucleus, characterized by the same local density of unbound neutrons. More precisely, in the approximation of uniform densities of the nucleus and the neutron gas, which is the same approximation underlying Eq. (3), we have $\Delta K_\xi = K(n_N) - K(n_G)$, where $K(n)$ is the kinetic energy of a sphere of radius R_N and particle density n, rotating rigidly with $\omega = \hbar/(2m_n \xi^2)$ and $\xi = \xi(n)$. An elementary calculation yields $K(n) = \pi \hbar^2 n R_N^5 / (15 m_n \xi^4)$. Since $\xi(n_N) \gg \xi(n_G)$ implies $K(n_G) \gg K(n_N)$, we finally obtain the following expression for the kinetic correction to the LDA model due to the quantum structure of the core

$$\Delta K_\xi \approx -\frac{\pi}{15}\frac{\hbar^2}{m_n} n_G \frac{R_N^5}{\xi_G^4}, \tag{4}$$

where ξ_G is the coherence length corresponding to the density n_G. In the following, we will take the coherence length corresponding to uniform density n as given by $\xi(n) = \frac{\hbar v_F}{\sqrt{6}\Delta(n)}$ with $v_F = \hbar k_F/m_n$. The factor $1/\sqrt{6}$, instead of the more standard $1/\pi$, follows from defining the coherence length as the distance from the vortex axis where the kinetic energy density ε_{kin} is equal to the pairing condensation energy density ε_{con}, consistently with what discussed in Paper I.

We notice that the schematic uniform-density model for pinning, developed in Ref. [11], is a reasonable approximation of the realistic-density LDA model, as can be seen by comparing the results of Refs. [11] and [12]. For this reason the terms ΔK_N and ΔK_ξ, both calculated in the uniform-density approximation, are compatible with our realistic model and should provide at least the correct order of magnitude of these additional kinetic contributions to pinning.

3. Results of the model

We can now show the numerical results of the calculation. We proceed exactly as in Paper I, except that we use Eq. (1) with $\alpha = 0.5$ for the Fermi internal energy and Eq. (2) with either $\beta = 2$ or $\beta = 3$ for the pairing gap. We skip most details and only present some relevant quantities to be compared to the results of Paper I for the Argonne gap, which would correspond to the choice $\alpha = \beta = 1$.

In Table 2 we report the Fermi energy μ calculated as in Paper I for each zone and for different values of the parameters α and β. The effect of introducing α is to reduce by about one half the chemical potential, that is the overall energy scale. The strong dependence of μ on the value of α shows how important it is to go beyond the free Fermi gas model. On the other hand, μ is much less dependent from β, i.e. on the value of the pairing gap, which is to be expected since pairing represents only a small correction to the total nuclear energy and which is an advantage here, when considering the present theoretical uncertainty of β. With $\alpha = 0.5$, the relation between neutron density and Fermi energy expected for the interacting system is approximately recovered, as can be seen (at least for the lower density zones) by comparing our results with Figs. 1 and 2 of Ref. [21], where low-density neutron matter is studied in the energy density functional approach.

Table 2
Fermi energies, μ, of the five zones calculated for different values of the parameters α and β, and given in MeV

Zone		1	2	3	4	5
$\alpha = 1$	$\beta = 1$	1.02	5.45	14.32	24.82	36.79
$\alpha = 0.5$	$\beta = 1$	0.42	2.58	7.15	12.46	18.42
$\alpha = 0.5$	$\beta = 2$	0.55	2.80	7.16	12.39	18.40
$\alpha = 0.5$	$\beta = 3$	0.58	2.84	7.17	12.37	18.39

Proceeding as in Paper I, we consider the nuclear pinning (NP) configuration (vortex through the nucleus) and the interstitial pinning (IP) configuration (vortex equidistant between two nuclei). Again, we find that the 'mixed' phase (vortex with core of normal matter) is always energetically favored over the 'pure' phase (vortex with empty core). We then evaluate the difference in energy between the NP and the IP configurations for each zone, taking always the 'mixed' structure for the vortex. In Table 3 we show the results for the difference in total energy $\Delta E_{\text{tot}} = E_{\text{tot}}^{(\text{NP})} - E_{\text{tot}}^{(\text{IP})}$ calculated for different values of the parameters α and β. The energy differences between the two configurations, ΔE_{tot}, are sensitive to both the parameters α and β. Altogether, the values of $|\Delta E_{\text{tot}}|$ are significantly *reduced* by the two corrections introduced so far.

We finally discuss the kinetic corrections to the LDA approach, starting with the nuclear contribution, ΔK_N. We remind that in Eqs. (3) and (4) the kinetic contributions were defined as $\Delta K = K^{(\text{NP})} - K^{(\text{IP})}$, consistently with the other differences of energy. As already discussed, in Paper I we introduced the cutoff $R < R_{\max}$ ($R_{\max} = R_{\text{WS}} - R_N$) when integrating on the (cylindrical) radial coordinate R. This implies that in the IP configuration the presence of the nucleus at a distance D from the vortex was neglected, and we want to correct this by using Eq. (3). Looking at Fig. 1 of Paper I we should take $D = R_{\text{WS}}$ in Eq. (3), but deeper in the crust the situation is actually more complicated. In fact, when reducing the gaps by a factor β, the corresponding coherence lengths get larger by the same factor. In Table 4 we give for each zone the coherence length, $\xi_G = \xi(n_G)$, associated to the neutron density n_G and to the pairing gap $\Delta(n_G)|_\beta$. As discussed in Paper I, to a good approximation ξ_G corresponds to the radius of the vortex core in the IP configuration, since away from the nucleus the density is just n_G. Now, in zones 1 to 3 we have $\xi_G < R_{\max}$, that is the vortex core is within the cutoff, so that we can apply Eq. (3) with $D = R_{\text{WS}}$. In zones 4 and 5, however, not only ξ_G is larger than the cutoff, but we actually have $\xi_G > R_{\text{WS}}$, that is the vortex core is larger than the Wigner–Seitz cell. In such a case, the distinction between nuclear and interstitial pinning has no more meaning, since the vortex core now contains several nuclei. We

Table 3
Difference in total energy, $\Delta E_{\text{tot}} = E_{\text{tot}}^{(\text{NP})} - E_{\text{tot}}^{(\text{IP})}$, between the NP and IP configurations calculated for the five zones and for different values of the parameters α and β, and given in MeV

Zone		1	2	3	4	5
$\alpha = 1$	$\beta = 1$	2.63	1.55	−5.21	−5.06	−0.35
$\alpha = 0.5$	$\beta = 1$	2.39	3.16	−4.09	−8.10	−0.35
$\alpha = 0.5$	$\beta = 2$	0.81	0.46	−2.71	−1.64	−0.06
$\alpha = 0.5$	$\beta = 3$	0.21	0.37	−2.30	−0.63	−0.02

Table 4
Coherence length, ξ_G, corresponding to the neutron density n_G and to the pairing gap $\Delta(n_G)|_\beta$, calculated for each zone and given in fm

Zone	1	2	3	4	5
$\beta = 2$	13.4	8.7	10.3	22.3	77.6
$\beta = 3$	20.0	13.0	15.4	33.5	116.4

will call this configuration *collective* pinning. It was already noticed and discussed in Ref. [8]: we will come back to this at the end of the section.

We can still estimate the pinning properties of zones 4 and 5. Also in the case of collective pinning, the pinning energy per site is associated to the change of energy when a nucleus enters or leaves the core (this fact was also mentioned in an Appendix of Ref. [8]). Indeed, let us consider *only* the nuclei which lie in the plane with origin in the pinning site and perpendicular to the vortex axis, and are contained in the circle $\mathcal{C}_G$ of radius ξ_G representing the transverse section of the core with the plane. Notice that, with respect to the 3-dimensional lattice, the microscopic pinning energies *per site* are equivalent to a 2-dimensional problem (as if there were no nuclei out of the plane we are considering). The third dimension (the one along the vortex axis, where the vortex pins to several sites in the lattice lying on different planes) enters in the pinning energies *per unit length*, which are the ones relevant at the macroscopic level; we will come back to this at the end of the section. Now, if the addition of a nucleus lowers the system energy (as it is the case for zones 4 and 5), at equilibrium the vortex core will position itself in order to maximize the number of nuclei contained in $\mathcal{C}_G$ and it will be pinned in this configuration. By moving the vortex core it is possible to go to a higher energy configuration with one nucleus less in $\mathcal{C}_G$, and thus effectively unpin the vortex from its equilibrium position and move it away from the pinning site. We next remind that, when $\xi_G > R_{\max}$, our LDA calculation simply sets to zero the kinetic term, namely $\varepsilon_{\rm kin} = 0$. Therefore the values of $\Delta E_{\rm tot}$ reported in Table 3 for zones 4 and 5 actually represent the change in energy (internal plus condensational) when a superfluid nucleus outside the large vortex core is moved inside it and decondenses to normal matter. Of course, to get the complete pinning energy per site we still have to account for the kinetic energy lost by the nucleus when it leaves the superfluid flow and enters the normal core: this is given by Eq. (3). Since the periodicity of the nuclear lattice is $2R_{\rm WS}$, we expect that on the average (for random vortex-lattice orientations) the vortex has to move by $\approx R_{\rm WS}$ to go from the position with one nucleus less inside the core to the equilibrium configuration, where that nucleus has just entered the core and is within a distance ξ_G from the axis; the distance between the vortex axis and the nucleus in its initial outside position will then be $D \approx \xi_G + R_{\rm WS}$.

Summarizing, to calculate ΔK_N we will use Eq. (3) with $D = R_{\rm WS}$ in zones 1 to 3 and with $D = \xi_G + R_{\rm WS}$ in zones 4 and 5. As for the kinetic contribution from the quantum core, ΔK_ξ, it can be easily calculated from Eq. (4) using the coherence lengths of Table 4. In Table 5 we show the values of the two kinetic corrections obtained with the previous prescriptions, as well as their sum $\Delta K_{\rm tot} = \Delta K_N + \Delta K_\xi$. The argument yielding Eq. (4) does not apply in the collective pinning configuration and therefore in Table 5 the corresponding values (which are anyways very small) are indicated in parenthesis and not included in the total. Altogether, the total kinetic correction is not negligible in the central part of the inner crust (zones 2 to 4), particularly so in zone 3.

Table 5
Kinetic corrections due to the nucleus, ΔK_N, and to the quantum core, ΔK_ξ, and total kinetic correction $\Delta K_{\rm tot} = \Delta K_N + \Delta K_\xi$, calculated with Eqs. (3) and (4) and given in MeV. The values in parenthesis correspond to collective pinning and they are not included in the total

Zone		1	2	3	4	5
$\beta = 2$	ΔK_N	-10^{-3}	−0.06	−0.38	−0.15	-10^{-3}
	ΔK_ξ	-10^{-3}	−0.10	−0.29	(−0.02)	(-10^{-4})
	$\Delta K_{\rm tot}$	-10^{-3}	−0.16	−0.67	−0.15	-10^{-3}
$\beta = 3$	ΔK_N	-10^{-3}	−0.06	−0.38	−0.09	-10^{-3}
	ΔK_ξ	-10^{-4}	−0.02	−0.06	(-10^{-3})	(-10^{-5})
	$\Delta K_{\rm tot}$	-10^{-3}	−0.08	−0.44	−0.09	-10^{-3}

Table 6
Difference in energy, $\Delta E = E^{\rm (NP)} - E^{\rm (IP)} = \Delta E_{\rm tot} + \Delta K_{\rm tot}$, calculated from Tables 3 and 5 and given in MeV. We also give the partial energy contributions (internal, condensational and kinetic) to ΔE

Zone			1	2	3	4	5
$\alpha = 0.5$	$\beta = 2$	ΔE	0.81	0.30	−3.38	−1.79	−0.06
		Internal	−0.06	0.04	−0.71	−0.26	0.02
		Condens	0.88	0.49	−1.98	−1.38	−0.08
		Kinetic	−0.01	−0.23	−0.69	−0.15	-10^{-3}
$\alpha = 0.5$	$\beta = 3$	ΔE	0.21	0.29	−2.74	−0.72	−0.02
		Internal	−0.16	0.16	−1.16	−0.01	0.01
		Condens	0.38	0.24	−1.12	−0.62	−0.03
		Kinetic	−0.01	−0.11	−0.46	−0.09	-10^{-3}

In order to obtain the complete energy difference responsible for pinning, the total kinetic correction must be added to the difference in total energy calculated in LDA with the cutoff and shown in Table 3; we remind that $\Delta E_{\rm tot}$ is obtained as the sum of three energy terms, that is $\Delta E_{\rm tot} = \Delta E_{\rm int} + \Delta E_{\rm cond} + \Delta E_{\rm kin}$ (cf. Paper I). In Table 6 we present the final results for $\Delta E = E^{\rm (NP)} - E^{\rm (IP)} = \Delta E_{\rm tot} + \Delta K_{\rm tot}$, corresponding to $\alpha = 0.5$ and to $\beta = 2$ or $\beta = 3$. We also give the partial energy contributions to ΔE, namely internal (Fermi plus nuclear), pairing condensation and total kinetic. The latter one, in particular, is $\Delta E_{\rm kin} + \Delta K_{\rm tot} \simeq \Delta K_{\rm tot}$ since $\Delta E_{\rm kin}$ turns out to be quite small for $\beta = 2$ or $\beta = 3$, due to the larger vortex core. Here too, we notice that none of the energy contributions to pinning is negligible in the central part of the inner crust (zones 2 to 4), particularly so in zone 3.

As in Paper I, $\Delta E > 0$ corresponds to interstitial pinning. In such a case, ΔE has no direct meaning, as the vortex can altogether avoid the nuclei by moving *around* them, and pinning is negligible: the effective force needed to unpin a vortex is orders of magnitude smaller than what found for nuclear pinning, namely $\sim 10^{-5}$–10^{-3} MeV/fm depending on the density [9]. On the other hand, $\Delta E < 0$ can correspond to either nuclear or collective pinning.

Before drawing our conclusions, we must clarify an issue relevant to glitch theory. In the present Letter we derive the pinning energy *per pinning site*, namely the microscopic interaction between a vortex and a single nucleus in its Wigner–Seitz cell. This amounts to considering the vortex-nucleus interaction limited to a single layer of nuclei contained in a plane perpendicular to the vortex axis. When actual vortex unpinning (and therefore glitches) is concerned, however, one needs the effec-

Table 7
Comparison of the pinning energies per site, E_P, and of the average pinning forces per site, F_P, calculated in this work with reduction factors $\beta = 2$ and 3, to those obtained in Paper I with the Argonne interaction and those obtained in Refs. [6,9] with the Landau–Ginzburg approximation. The energies are given in MeV, the forces in MeV/fm, while '–' indicates interstitial pinning (zones 1, 2) or collective pinning (zones 4, 5), which are both super-weak at the macroscopic level

	Zone	$\beta = 2$	$\beta = 3$	Paper I	Ref. [6]
E_P	1	–	–	–	–
	2	–	–	–	0.4
	3	3.4	2.7	5.2	15
	4	–	–	5.1	9
	5	–	–	0.4	5.4
F_P	1	–	–	–	–
	2	–	–	–	0.11
	3	0.13	0.10	0.19	3.6
	4	–	–	0.26	1.9
	5	–	–	0.03	1.7

tive energy necessary to unpin a macroscopic vortex from the lattice of nuclei, i.e. the pinning energy *per unit length*. This takes into account the several pinning sites (i.e., layers of nuclei) encountered by the vortex along its length and it needs a characteristic pinning length (average distance between pinning sites along the vortex) in order to be derived from the microscopic energies of Table 7. The authors of Ref. [8] argued convincingly that, when the core is larger than a WS cell, on average along the length of a moving vortex as many nuclei enter the core as they exit it, so that the pinning energy per unit length is actually very small: they called this regime 'super-weak pinning'. Therefore, collective pinning at the microscopic level implies super-weak pinning at the macroscopic level, irrespective of the actual value of ΔE. This is a welcome physical result for glitch theory, since the approach presented in this paper is a good approximation for the first three zones of the crust, while for zones 4 and 5 it is just a reasonable extrapolation.

Altogether, only nuclear pinning is relevant to pulsar glitches. In such a case, the pinning energy is $E_P = |\Delta E|$ and the *average* pinning force is given by $F_P = E_P / R_{\mathrm{WS}}$, since it corresponds to a displacement of the vortex by R_{WS}. As mentioned, these quantities must be used to calculate the corresponding parameters per unit length which appear in glitch models. In Table 7 we summarize our results for the pinning energies and the average pinning forces, and compare them with those from the LDA approach in Paper I (we only quote the values for Argonne) and those from the Landau–Ginzburg approach in Ref. [6]. Due to the intrinsic uncertainties of the treatment, we round off energies to one decimal digit and forces to two decimal digits, while '–' indicates interstitial pinning (zones 1, 2) or collective pinning (zones 4, 5), both negligible at the macroscopic level independently from ΔE.

We observe that the values of the pinning energies and the average pinning forces obtained here are significantly smaller than those calculated in Paper I, where $\alpha = \beta = 1$ and $\Delta K_{\mathrm{tot}} = 0$. In turn, the results of Paper I were already much smaller than those from the earlier Landau–Ginzburg approach. As one would expect, larger values of β, corresponding to more in-medium suppression of pairing, yield weaker pinning parameters.

In conclusion, the only region presenting significant pinning at the macroscopic scale (i.e., nuclear pinning) is zone 3, while lower densities correspond to interstitial pinning and larger densities to super-weak pinning, both macroscopically negligible. Moreover, when compared to previous treatments, the pinning energies and average forces per site are quite weak, as generally required by observations of pulsar glitches. Finally, the uncertainty range of the pairing reduction factor, β, yields for the pinning energies in zone 3 a range of uncertainty $2.5 < E_P < 3.5$ MeV, significant but presently unavoidable.

4. Conclusion

In this Letter we have obtained stronger limits on the pinning properties of vortices in the inner crust of neutron stars, required to test the vortex theory for pulsar glitches. We have used our semi-classical model of Paper I, to date the most predictive approach, and corrected some of its energy terms in order to reproduce known properties of neutron matter at intermediate densities, to account for medium polarization effects on the gap and to better estimate the kinetic energy of the vortex–nucleus system, also accounting for the quantum structure of the vortex core.

The main conclusion of the Letter is a *restricted* density range (about $2 \times 10^{13} \lesssim \rho \lesssim 5 \times 10^{13}$ g/cm^3 or $0.07\rho_0 \lesssim \rho \lesssim 0.2\rho_0$) of *weak* nuclear pinning ($E_P < 3.5$ MeV), the only configuration yielding significative pinning of vortices at the macroscopic scale relevant to pulsar glitches. The rest of the crust presents either interstitial pinning (for $\rho < 0.07\rho_0$) or collective super-weak pinning (for $\rho > 0.2\rho_0$), both situations corresponding to negligible effective pinning.

We believe that our approach is altogether physically reasonable and consistent, and that the results obtained here are more realistic than those existing in the literature. The main uncertainty in the pinning energies (i.e., the range $2.5 < E_P < 3.5$ MeV for zone 3, the physically relevant region of nuclear pinning) is related to the parameter β, but as long as the in-medium suppression of pairing is not fully understood, any reasonable calculation of pinning properties can *only* yield an interval of values for E_P.

In the thirty years since the vortex theory was proposed [2], successive improvements in the microscopic approach resulted in a weakening of the calculated pinning. One may wonder whether in a *consistent* quantum calculation the results obtained here will be mostly confirmed, albeit possibly further reduced by proximity effects [13], or they will be significantly altered: we believe that the first alternative is more likely. In the meantime, it would certainly be instructive to use these weaker values for the vortex–nucleus interaction in macroscopic models of pulsar glitches and to compare the results with observational constraints. We are currently studying a realistic model to obtain the critical difference of angular velocity for vortex unpinning, $\Delta\Omega_{\mathrm{cr}}$, from our microscopic pinning energies.

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