Effective Nucleon Mass in Deformed Nuclei

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The coupling of vibrations to nucleons moving in levels lying close to the Fermi energy of deformed rotating nuclei is found to lead to a number of effects: (i) shifts of the single-particle levels of the order of 0.5 MeV towards the Fermi energy and thus to an increase of the level density, (ii) single-particle state depopulation of the order of 30%, and thus spectroscopic factors $\sim$0.7, etc. These effects, which we have calculated for ^{168}Yb, can be expressed in terms of an effective mass, the so-called ω mass (m_ω), which is $\approx$40% larger than the bare nucleon mass in the ground state. It is found that m_ω displays a strong dependence with rotational frequency, eventually approaching the bare mass for $\hbar\omega_{\rm rot} \approx 0.5-0.6$ MeV.

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Mean-field theory is one of the most useful approximations in all of physics. In it, one replaces the many-particle Schrödinger equation by a single-particle Schrödinger equation, describing the independent-particle motion of nucleons in an average potential. In Hartree-Fock approximation, there exist two contributions to this potential: a local contribution (Hartree term) and a nonlocal contribution (Fock term), whose origin can be traced back to the fact that nucleons are fermions obeying the Pauli principle, aside from that arising from a possible momentum dependence of the nucleon-nucleon effective interaction. The nonlocality of the mean-field potential is equivalent to a dependence of the potential on the linear momentum of the particle. This dependence can, for many purposes, be accounted for by an effective mass, the so-called k mass m_k which, as a rule, is considerably smaller than the bare nucleon mass m (cf., e.g., [1] and references therein). Consequently, Hartree-Fock theory is able to predict the sequence of levels of single-particle states around the Fermi energy, but not the level density. This limitation is connected with the fact that one is dealing with a static approximation to the many-body problem. In fact, the mean field defines a surface which can fluctuate. These fluctuations and the associated renormalization effects on nucleon properties like mass, charge, etc., are essential ingredients of mean field theory. No description of the single-particle motion can be considered complete without explicitly taking the corresponding particle-vibration coupling phenomena into account, which is indeed one of the most striking features of finite quantum systems.

The coupling of the nucleons to the vibrations of the nuclear surface leads to a contribution to the single-particle self-energy which is nonlocal in time. In other words, a term which depends on the energy $\hbar\omega$ of the single-particle states. This term can again be absorbed, quite accurately, in an effective mass, the so-called ω mass m_ω, which is considerably larger than the bare nucleon mass m [1]. The net result of absorbing the nonlocal effects associated with the Fock term (plus an eventual contribution from the velocity dependent terms of the two-body interaction) and with the surface induced self-energy into the kinetic energy term, leads to single-particle motion where nucleons of effective mass $m^* = m_k m_\omega / m \approx m$ move in a local potential. Although this picture is deceivingly similar to the phenomenological picture worked out to describe the experimental evidence of the single-particle spectrum of levels lying close to the Fermi energy, where nucleons of mass m move in a Saxon-Woods potential, essential differences with the effective mass picture are to be noted. In particular, the empirical Saxon-Woods picture posits that the spectroscopic factors of occupied single-particle levels, including those close to the Fermi energy, are equal to one. However, in keeping with the fact that the average spectroscopic factor associated with these levels is given by $Z_\omega = (m_\omega/m)^{-1}$ and that $m_\omega/m > 1$, the effective mass model will predict a sizable depopulation for these levels. In other words, a nucleon moving close to the Fermi surface will spend, on average, a fraction $(1 - Z_\omega)$ of its time in configurations $(2p\text{-}1h, 3p\text{-}2h, \ldots)$ other than single-particle configurations. This result is expected to have important consequences for the low-energy behavior of the system, for example, concerning the ability with which levels close to the Fermi energy can contribute to form Cooper pairs and thus to sustain a superfluid phase.

While calculations of the effective ω mass have been carried out in spherical nuclei both at zero as well as at finite temperature (cf. Refs. [1–10], and references therein), also making use of effective forces and self-consistent methods [11,12], no single calculation exists of this quantity in deformed, let alone rotating nuclei. This is the subject of the present paper, where we present, for the first time, a calculation of the nucleon ω mass for a medium-heavy deformed nucleus as a function of the rotational frequency along the yrast line [13]. It will be concluded

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that the ω mass in deformed nuclei is about 40% larger than the bare mass, a result similar to that found in the case of the spherical tightly bound nucleus ^{208}Pb [1], and smaller than the values $m_\omega/m = 1.7-1.8$ obtained for superfluid spherical nuclei [4,5], due to the fact that, as a rule, surface vibrations are, in deformed nuclei, less collective than in spherical nuclei. The ratio m_ω/m is found to approach one, as a function of $\omega_{\rm rot}$, a phenomenon which is correlated with the increased rigidity of the nuclear surface due to the polarization effects of the centrifugal force at higher rotational frequencies.

The calculations have been carried out in a cranked Nilsson single-particle basis labeled by parity $\pi = \pm$ and signature quantum number $\alpha = \pm 1/2$. The quasiparticle energies have been calculated in the BCS approximation, making use of this basis and of a monopole pairing interaction with constant matrix elements. The residual separable interactions of strength χ_ρ^α describing both quadrupole and octupole modes were diagonalized in the random phase approximation (RPA). The signature quantum number for collective operators is $\alpha = 0, 1$ (see [15] for details).

As compared to the case of spherical nuclei, the vibrational modes in rotating, deformed nuclei are strongly fragmented over many, closely spaced, two-quasiparticle-like states, due to the lower symmetry the system has as compared to a spherical nucleus. A detailed characterization of the thousands of RPA roots will thus be both numerically taxing and not very useful. This is the reason why we have rewritten the corresponding RPA equations in terms of the linear response function of the system, and solved them making use of a coarse grain approximation.

A second problem encountered in connection with the collective response of the system, as compared to the spherical case, is the presence of spurious states associated with the violation of rotational invariance. Furthermore, the spurious states associated with the violation of particle number, typical of BCS solutions, have also to be eliminated from the vibrational spectra (cf., e.g., [16,17]), because, in the case of deformed nuclei, pairing vibrations [18], that is vibrations which change the number of particles by two, couple to quadrupole vibrations around the symmetry axis, so called β vibration ($K = 0$) [19]. This elimination is accomplished making use of complex integration techniques (cf. [20], and references therein). The resulting RPA solutions provide not only the poles and residues (energies and transition amplitudes) of the collective and noncollective quadrupole and octupole modes of the system, but also the quasiparticle-vibration coupling vertices.

Making use of all these elements, the single-particle self-energy processes shown in the inset to Fig. 1 were calculated in terms of the following expression containing integrals over the response functions (see [15,21] for details),

$$\Sigma_{\rm ret}(\xi, E + iI) = \sum_\eta \sum_{\alpha,\rho\rho'} q_{\rho\rho'}^{(\alpha)}(\xi\eta) \Bigg[\frac{1}{2}\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\mathcal{R}_{\rho\rho'}^{(\alpha)}(i\omega) + \mathcal{R}_{\rho'\rho}^{(\alpha)}(i\omega)}{\bar{E}_\eta - i\omega} + \frac{s_{\xi\eta}^{(\alpha)}}{2}\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\mathcal{R}_{\rho\rho'}^{(\alpha)}(i\omega) - \mathcal{R}_{\rho'\rho}^{(\alpha)}(i\omega)}{\bar{E}_\eta - i\omega} - \begin{cases} \frac{s_{\xi\eta}^{(\alpha)}+1}{2}\mathcal{R}_{\rho\rho'}^{(\alpha)}(\bar{E}_\eta), & \text{if } E > s_{\xi\eta}^{(\alpha)}E_\eta \\ \frac{s_{\xi\eta}^{(\alpha)}-1}{2}\mathcal{R}_{\rho'\rho}^{(\alpha)}(\bar{E})_\eta, & \text{if } E < s_{\xi\eta}^{(\alpha)}E_\eta \end{cases} \Bigg], \quad (1)$$

where, for simplicity, we have introduced the notation $\bar{E}_\eta = E + iI - s_{\xi\eta}^{(\alpha)}E_\eta$. The fact that the above expression is complex is associated with the conservation of signature quantum number at the vertices of the self-energy diagram drawn in the inset to Fig. 1 (see [21]) which gives rise to a phase, that is $s_{\xi\eta}^{(\alpha)} = (-)^{\alpha+\alpha_\xi-\alpha_\eta}$, where $\alpha_{\xi(\eta)}$ labels the signature of a quasiparticle states $\xi(\eta)$, while α is the vibrational signature index. In the above equation, the quantity $\Sigma_{\rm ret}(\xi, E + iI)$ indicates the retarded self-energy of the quasiparticle states ξ of energy $E + iI$, I being an average parameter (coarse mesh approximation) whose value is chosen equal to 0.5 MeV in all calculations. The quantity $q_{\rho\rho'}^{(\alpha)}(\xi\eta) \equiv \chi_\rho^{(\alpha)} R_\rho^{(\alpha)^*}(\xi\eta) R_{\rho'}^{(\alpha)}(\xi\eta)\chi_{\rho'}^{(\alpha)}$ contains the state involved at the vertices of the self-energy diagram (see inset to Fig. 1) labeled by ξ for the initial state and by η for the intermediate state, both of which take signature quantum number, $\alpha_{\xi(\eta)} = \pm 1/2$. The expression $R_\rho^{(\alpha)}(\xi\eta)$ represents the matrix elements of the different operators involved (distinguished by the index ρ for pairing, quadrupole, and octupole operators) expressed in the quasiparticle basis while $\mathcal{R}_{\rho\rho'}^{(\alpha)}(\omega)$ indicates the RPA linear response functions.

Calculations were carried out based on the yrast states of ^{168}Yb, for which an accurate description of the low-energy spectrum is available. The potential energy surface of this nucleus, calculated taking into account Nilsson-Strutinsky corrections, displays a stable minimum, with parameters $\epsilon_2 = 0.262$ and $\gamma = 0^0$ at the ground state, and approximately keeps its position up to the high-spin state, $I \approx 60\hbar$. Therefore, we have used the constant deformation parameters for convenience. The even-odd mass differences for neutrons and protons are used as the pairing gaps at zero frequency, which fix the monopole pairing force strengths. The doubly stretched quadrupole and octupole forces are used as residual interactions in the particle-hole channel. The quadrupole force strengths for

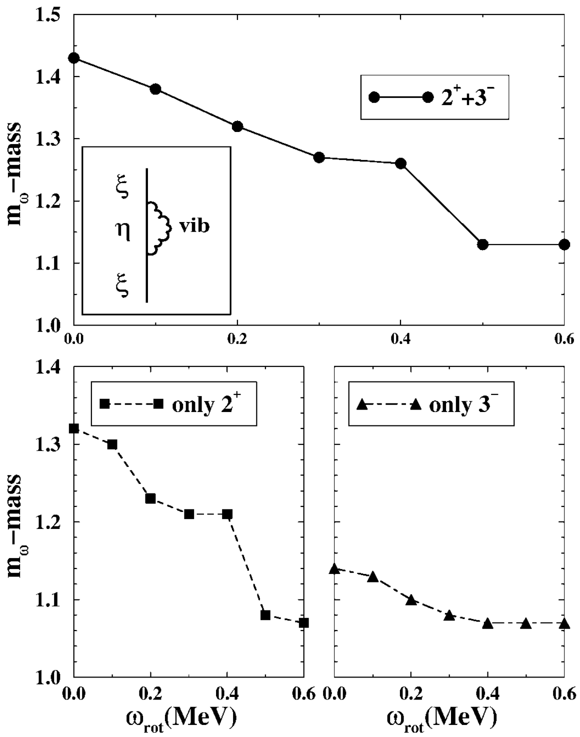

FIG. 1. ω mass m_ω as a function of the rotational frequency. The upper part of the graph contains the effects of the coupling to both quadrupole and octupole modes while the lower part displays the quadrupole (left panel) and octupole (right side) contributions, respectively. In the inset to the upper part of the figure is shown the basic diagram representing the self-energy of a quasiparticle state ξ. A wavy line stands for a surface vibration.

$K = 0$, 2, and 1 components are determined in such a way as to reproduce the excitation energies of the β and γ vibrations, and the spurious mode corresponding to the rotation (zero energy mode), respectively. As for the octupole strength, there are not enough experimental data to fix all the $K = 0-3$ components. We have used the self-consistent force strengths with a scaling factor, which is determined to give the lowest vibrational energy about 1.65 MeV. The model space used is $N_{\mathrm{osc}} = 3-8$ for neutrons and $N_{\mathrm{osc}} = 2-7$ for protons.

The resulting quasiparticle spectrum (routhians), that is, the eigenvalues of the cranked Nilsson plus mean field pairing Hamiltonian were calculated and, on the basis of two-quasiparticle states, the RPA linear response of the system determined. Making use of these elements, the self-energy [cf. Eq. (1)] of quasiparticles build on single-particle states lying around the Fermi energy have been calculated as a function of the rotational frequency. Because of the complexity of the calculations, in particular the conspicuous time-consuming elimination of the spurious states, we have restricted our calculations to 60 neutron quasiparticle states, all lying within an energy interval of 5 MeV from the yrast state in the rotational frequency interval considered, that is (0–0.6) MeV. The effective ω mass [1],

$$\frac{m_\omega(\xi)}{m} = 1 - \frac{d}{dE}\,\mathrm{Re}\Sigma_{\mathrm{ret}}(\xi, E + iI)\Big|_{E=\tilde{E}_\xi}, \quad (2)$$

can be calculated making use of the real part of the self-energy, and of the renormalized quasiparticle energies

$$\tilde{E}_\xi = E_\xi + \mathrm{Re}\Sigma_{\mathrm{ret}}(\xi, \tilde{E}_\xi + iI). \quad (3)$$

In Fig. 1 we show the average effective mass, as a function of the rotational frequency. In the upper part of the figure we display the results obtained summing up the effects of the quadrupole and octupole vibrations, while in the lower part of the figure we show the separate contributions.

It is seen that the coupling of particles moving in Nilsson levels lying close to the Fermi surface leads to an increase of $\approx 40\%$ of the nucleon effective mass, and thus to an energy shift of the order of 0.5 MeV towards the Fermi energy, increasing the level density. These levels have an average value of $Z_\omega \approx 0.7$, implying that a nucleon moving close to the Fermi energy spends, on average, 30% of the time in configurations other than single-particle configurations.

The result that the effective mass in deformed nuclei is smaller than in superfluid spherical nuclei, reflects the fact that the quadrupole collectivity of the surface modes in deformed nuclei is mainly used to build up the static deformation (spontaneous symmetry breaking phenomenon).

The decrease of the effective ω mass as a function of rotational frequency can be understood by analyzing the RPA response function, and thus the collectivity of the low-lying surface modes as a function of the rotational frequency. From Fig. 2 it is seen that the $K = 0$ and 2 quadrupole amplitudes for the low-lying modes (corresponding to the β and γ vibrations) decrease drastically (height of the peak at $\approx 1-1.5$ MeV), as a function of ω_{rot}, while the strength is spread over a wider energy interval. This behavior of the response function indicates a marked reduction of the collectivity of the low-lying modes as a function of the rotational frequency. This reduction is more conspicuous for the $K = 0$ component of the surface vibrational spectrum, a consequence, to a large extent, of the coupling of the β vibration to the pairing degrees of freedom. In fact, it is seen from the inset in the upper part of Fig. 2, where the BCS pairing gap is displayed as a function of the rotational frequency, that the system undergoes a phase transition from a superfluid to a normal phase, thus becoming more rigid against surface vibrations.

The calculations have been repeated introducing, in the Nilsson Hamiltonian, a bare mass $\frac{m_k}{m} = 0.76$, as in the case of a Skyrme III interaction. Although the resulting single-particle levels are not expected to coincide with those of a self-consistent constrained mean-field calculation of ^{168}Yb (not available in literature) based on an effective interaction (e.g., Skyrme III), the main features of a low-level density around the Fermi energy is found. In keeping with this result, the uncorrelated quadrupole and octupole response function are shifted upwards in energy

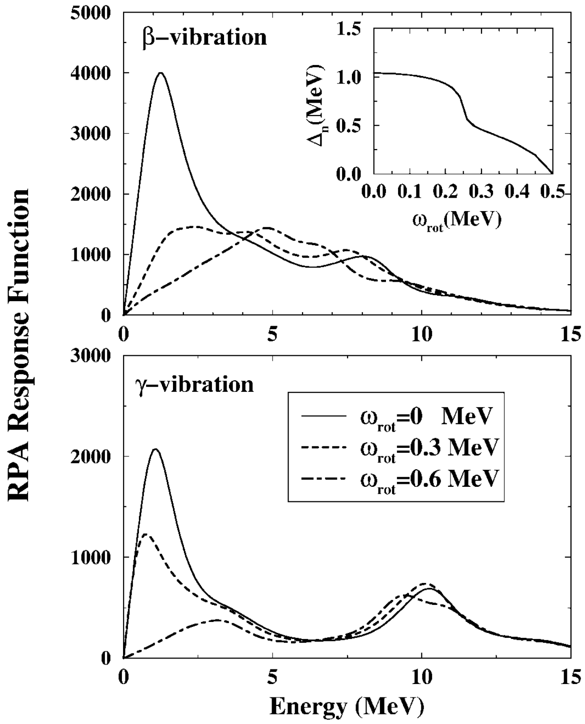

FIG. 2. Quadrupole RPA response function for ^{169}Yb as a function of the rotational frequency with averaging parameter $I = 0.5$ MeV. The ordinate is in units of $b_0^4/\hbar\omega_0$ where $\hbar\omega_0 = 41\mathrm{A}^{1/3}$ MeV and b_0 is the oscillator length. The upper panel shows the $K = 0$ response corresponding the β vibration, while the lower that of $K = 2$ corresponding to the γ vibration. The inset in the upper part displays the behavior of the neutron BCS pairing gap as a function of rotational frequency.

as compared to the (bare mass) Nilsson results. Properly adjusting the quadrupole and octupole force strengths to reproduce the main peaks of the correlated response functions displayed in Fig. 2, and repeating the calculations of Σ_{ret} one obtains an m_ω-effective mass which, for $\omega_{\mathrm{rot}} = 0$, is $\approx$20% larger than the value shown in Fig. 1. This result is understood in terms of the somewhat stronger collectivity displayed by β, γ, and octupole vibrations calculated with $\frac{m_k}{m} = 0.76$, as compared to those obtained with $\frac{m_k}{m} = 1$. The dependence of the $\frac{m_k}{m} = 0.76$ results is expected to be the same as that shown in Fig. 1, in keeping with the fact that the k mass is essentially independent on the rotational frequency.

We conclude that, in deformed nuclei, the coupling effects to the surface vibrations for nucleons moving in single-particle levels lying close to the Fermi energy leads to conspicuous effects—in particular, to energy shifts of the order of 0.5 MeV and to single-particle depopulation of the order of 30%. These results will force us to review the single-particle description of deformed nuclei close to the yrast line (cf., e.g., [22] and references therein). Furthermore, because of the marked dependence these effects have on the rotational frequency, we expect they should help, among other things, to get a better description of the pairing phase transition expected to occur in deformed rotating nuclei as a function of spin, and for which much indirect evidence has been accumulated (cf., e.g., Ref. [23], and references therein).

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