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Nucleon–phonon coupling in hot, rapidly rotating nuclei (I)

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Abstract

The particle–vibration coupling formalism is generalized to deal with hot, strongly rotating nuclei. In this way the basis to carry out a microscopic description of the structure of compound nuclei has been developed, in particular the calculation of the effective mass of nucleons close to the Fermi energy. As a consequence, it will be possible to provide a realistic estimate of the nuclear level density as a function of rotational frequency and temperature, a quantity which is among the key ingredients needed in the study of the evolution of the cooling of the compound nucleus through nucleon and γ-decay. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

In mean field theory nucleons move independently of each other filling the combined pullings and pushings of the other nucleons when trying to leave the nucleus. The nuclear surface is not only a static barrier confining self-consistently the motion of the nucleons, but can be excited by them into vibrational modes which can, at a later time, be reabsorbed by the same or by a different nucleon. Associated with these phenomena are the mean free path (cf. e.g. Ref. [1] and references therein), nucleon effective mass

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(ω-mass) (cf. e.g. Ref. [2] and references therein) and the nucleon–nucleon induced interaction (cf. Refs. [3–5] and references therein).

While much effort has been concentrated in the study of the particle–vibration coupling and its consequences for the structure of spherical nuclei (cf. e.g. Refs. [6–8] and references therein), little has been done concerning deformed nuclei let alone hot, rotating systems. This situation is not a particularly brilliant, if one thinks that (a) to extract information from the decay of compound nuclei (i.e systems corresponding to a statistical ensemble of shapes) which can be directly compared to model predictions one needs to know the level density as a function of temperature and of angular momentum, a quantity which depends, through the level density parameter, on the nucleon effective mass and thus on the ability the particles have to couple to surface vibration in deformed nuclei, (b) to accurately calculate the alignment of single-particle levels, a quantity needed in the study of rotating nuclei in general, and the study of the superfluid-normal nuclear phase transition as a function of the angular momentum and of temperature in particular, as well as in the study of the damping of rotational motion, one needs to calculate the coupling of single-particle motion to pairing vibrations [9–11] and thus also to surface vibrations of deformed nuclei, (c) to determine the properties of the levels of a cranked Hartree–Fock Hamiltonian entering in the calculation of the intrinsic states of rotational bands as well as in the calculation of giant resonances in general, and of the giant dipole resonance in particular [12], one needs to know both the single-particle effective mass and width, and thus its coupling to surface vibrations of deformed nuclei.

To qualitatively change the level of accuracy with which the variety of quantities are calculated going beyond mean field, one needs to take up the task to reformulate the particle–vibration coupling model which lies at the basis of Nuclear Field Theory, for the case of superfluid, strongly deformed, rapidly rotating and highly excited nuclei. This is the main goal we have set us, and on which we shall further elaborate in this introduction.

For both spherical and deformed nuclei, a field theoretical treatment of the interweaving of single-particle and collective degrees of freedom exist (cf. Refs. [4,13,14] and references therein). However, while theory has been developed into a powerful predictive tool in the case of spherical systems and the associated computational scheme has been tested in a large variety of situations and used with success in the analysis of large class of experiments, essentially no calculations exist in the literature for deformed nuclei. The reason for such asymmetry in the study of the particle–vibration phenomenon is quite simple and is associated with the special role spurious states play in deformed nuclei as compared to spherical nuclei.

In fact, within mean field theory of deformed, superfluid nuclei, there are two classes of spurious states. One is associated with the violation of rotational invariance, the other with the violation of gauge invariance. In other words, because the BCS wave function in the Nilsson basis has neither a fixed value of the angular momentum nor of the number of particles, the system described by this wave function displays modes with vanishing frequency and divergent matrix elements of the quadrupole and of the two-

particle transfer operators. These spurious states, together with that associated with the violation of translational invariance typical of the shell model, can be made orthogonal to all physical states by diagonalizing the corresponding particle–vibration coupling Hamiltonian, in the random phase approximation (cf. e.g. Ref. [4] and references therein).

However, when a particle is added to the system, the spurious states, through the coupling to the odd-nucleon, mix again into the spectrum. The solution to this problem is a necessary condition to be able to carry out nuclear structure calculations in deformed, superfluid nuclei which go beyond mean field theory. While in the case of spherical nuclei, the spurious state associated with the nonconservation of the number of particles typical of superfluid nuclei does not mix, as a rule, with surface vibrational modes (quadrupole, octupole, multipole vibrations) due to angular momentum conservation, in deformed nuclei, the pairing spurious modes do mix with surface modes. This is the reason why, in spite of the fact, as mentioned before, that a field theoretical treatment of the particle–vibration phenomenon in deformed nuclei exists, it still awaits a practical formulation. On the other hand, in a previous paper [15], the authors have given an operative and economic solution to the question of how to deal with spurious states in superfluid, deformed nuclei, which have been tested both against schematic models for which exact solutions exist, as well as against the experimental finding through detailed applications to realistic situations [16]. In this sense, Ref. [15] can be viewed as the first of a trilogy of which the present paper is the second and Ref. [16] will be the third.

In keeping with the discussion above, it is now easy to explain the rationale lying behind the present paper. Once one knows how to eliminate the spurious states in an arbitrary situation, one needs to implement the calculations of the nuclear spectrum and of the associated transition probabilities, taking also into account the particle–vibration coupling phenomenon. Because of the lower symmetry displayed by mean field solutions describing the system as compared to the original Hamiltonian, both the single-particle and the collective spectrum of deformed, superfluid nuclei are richer than that observed around closed shell nuclei. If one furthermore sets the nucleus into rotation, time reversal invariance is also violated and the spectrum becomes even richer.

From the point of view of computation, richness implies complication. In particular, to find the energy of the vibrational modes which couple to the motion of the nucleons becomes, assuming the spurious states have been already removed from the spectrum, essentially an impossible task. In particular, within RPA, and because both particle–hole and particle–particle channels with both angular momentum $L = 0$ and 2 mix (e.g. pairing and β-vibration), the explicit determination of each single root of the RPA matrix is out of the question. This is because one is dealing, essentially, with a continuous of vibrational modes to which the nucleus couples, a situation which does not become easier to handle when temperature is added to the system.

Similar problems have been encountered in the study of infinite systems. However, this case is qualitatively simpler than the nuclear case, in the sense that the properties of the single-particle spectrum change smoothly with energy, not displaying the strong

state dependence typical of deformed rapidly rotating nuclei, a dependence which is the fingerprint of the finite character of the system (so-called quantal size effects (QSE), as they were first called by Kubo in his seminal studies of small particles [17,18]). In fact, one has to consider an infinite system with impurities (also of magnetic type which parallel the high-j intruder states found in the case of the nuclear spectrum) leading e.g. to Anderson localization and to high T_c (cf. e.g. [19,20]) to come close to the taxing computational situation one encounters in dealing with the particle–vibration phenomenon in deformed, superfluid, rapidly rotating nuclei.

In spite of all these difficulties, and as argued above, the development of a formalism to treat the particle–vibration coupling in warm, strongly rotating nuclei is a task which can hardly be delayed, and a proper analysis of the variety of phenomena which are at the basis of the nuclear structure of these systems is long overdue. A task which is at the basis of the results presented in this paper and which, in spite of being highly technical (determination of the quantum numbers characterizing the single-particle basis, formulation of Matusbara's finite temperature Green's function to describe the particle–vibration phenomena in terms of linear response theory taking into account QSE, accidental degeneracies and thus zero-energy denominators, etc.) results in simple, powerful expressions (like that provided by Eqs. (23), (25) and (27) for the effective mass) to calculate the observables of the system and thus the corrections to mean field theory, creating dressed particles (quasiparticles) whose properties can be directly confronted with the experimental findings.

At the basis of the formulation of the particle–vibration coupling developed in the present paper is the idea, also used in condensed matter physics, to recur to linear response function theory to deal with the particle–hole and particle–particle (two-quasiparticle like) excitation spectrum of the system. Computationally, this is to a large extent equivalent to use a course mesh approach for the calculation of the vibrational spectrum and its coupling to the nucleons.

In what follows we developed the formalism describing the particle–vibration coupling in highly excited, deformed, rapidly rotating nuclei. In Section 1 the quasiparticle basis is introduced, while surface and pairing vibrations are discussed in Section 2 within the framework of the random phase approximation (RPA) in a two quasiparticle basis. The interweaving of the quasiparticle motion and the collective modes at zero and at finite temperature is discussed in Section 3 making use of Matsubara's formalism (cf. Ref. [21]). The last section is devoted to the conclusions. The results of the application of this formalism will be presented in part II of this paper [16].

2. Quasiparticle in a rotating frame

The starting point of the calculation is the Hamiltonian

$$\hat{h}_{\rm rot} = \hat{h}_{\rm def} + \hat{U}_{\rm P} - \lambda\hat{N} - \omega_{\rm rot}\hat{J}_x\,, \tag{1}$$

sum of a Nilsson Hamiltonian $\hat{h}_{\rm def}$ [22] and of the BCS pairing mean field

$$\hat{U}_{\mathrm{P}} = -\Delta(\hat{P}^{\dagger} + \hat{P}) , \tag{2}$$

where $\hat{P}^{\dagger} = \sum_i \hat{c}_i^{\dagger}\hat{c}_{\tilde{i}}^{\dagger}$ is the pair creation operator. The single-particle state $\tilde{i}$ is time reversed to the single-particle state i, while Δ is the pairing gap. To restore rotational and gauge invariance, at least in average, the system is set in rotation through the external fields (cranking approximation) $-\omega_{\mathrm{rot}}\hat{J}_x$ and $-\lambda\hat{N}$, where $\hat{J}_x$ and $\hat{N}$ are the angular momentum and particle number operators, respectively, while ω_{rot} and λ are rotational frequencies in 3D- and in gauge-space. Appropriately adjusting the value of these parameters, which can be viewed as Lagrange multipliers of the energy minimization problem (diagonalization of $\hat{h}_{\mathrm{rot}}$), one can fix the average number of particles and the average value of the angular momentum. In keeping with the fact that $\hat{h}_{\mathrm{rot}}$ is invariant only with respect to a rotation of angle π around the x-axis, and with respect to parity transformation, the quantum numbers which identify the corresponding single-particle eigenstates are signature α (see Appendix A for details) and parity π. We shall denote with μ single-particle states with signature negative $\alpha = +1/2$ $(r = -i)$ and with $\bar{\mu}$ positive single-particle signature states $\alpha = -1/2$ $(r = +i)$. Within the basis of states $|\alpha, \pi\rangle$ the Hamiltonian $\hat{h}_{\mathrm{rot}}$ is diagonalized by rewriting it in term of quasiparticle $\hat{a}_{\mu}^{\dagger}$ operators (cf. Ref. [23]), leading to

$$\hat{h}_{\mathrm{rot}} = E_0 + \sum_{\mu} E_{\mu}\hat{a}_{\mu}^{\dagger}\hat{a}_{\mu} + \sum_{\bar{\mu}} E_{\bar{\mu}}\hat{a}_{\bar{\mu}}^{\dagger}\hat{a}_{\bar{\mu}} , \tag{3}$$

where $E_{\mu(\bar{\mu})}$ is the quasiparticle energy associated with the state $\mu(\bar{\mu})$.

3. The RPA in a rotating frame

In what follows we shall calculate the variety of vibrational nuclear modes in deformed rotating nuclei by diagonalizing, in the RPA and in a two-quasiparticle basis, a monopole pairing interaction and multipole–multipole particle–hole interactions, that is,

$$\hat{H}_{\mathrm{int}}^{(\alpha)} = -\tfrac{1}{2}\sum_{\alpha}\sum_{\rho}\chi_{\rho}^{(\alpha)}\left\{\hat{R}_{\rho}^{(\alpha)}\right\}^2 , \tag{4}$$

where $\hat{R}_{\rho}^{(\alpha)}$ are one-body Hermitian operators, and $\chi_{\rho}^{(\alpha)}$ the corresponding coupling strengths. The indices α and ρ represent, respectively, the signature quantum number ($\alpha = 0, 1$ labeling positive and negative signatures (cf. Appendix B)), and the different components of residual interactions. The coupling strengths χ should be determined by the self-consistent condition between the density distribution and the single-particle potential. As explained above, we adopt the following operators:

$$\begin{aligned}
\hat{R}_{\rho}^{(0)} &= \tilde{P}_{n+} \quad \tilde{P}_{n-} \quad \tilde{P}_{p+} \quad \tilde{P}_{p-} \quad \tilde{Q}_{20}^{(0)} \quad \tilde{Q}_{21}^{(0)} \quad \tilde{Q}_{22}^{(0)} \quad (\pi = +) , \\
&= \tilde{Q}_{31}^{(0)} \quad \tilde{Q}_{32}^{(0)} \quad \tilde{Q}_{33}^{(0)} \qquad\qquad\qquad\qquad\qquad\qquad (\pi = -) , \qquad (5) \\
\hat{R}_{\rho}^{(1)} &= \tilde{Q}_{21}^{(1)} \quad \tilde{Q}_{22}^{(1)} \qquad\qquad\qquad\qquad\qquad\qquad (\pi = +) , \\
&= \tilde{Q}_{30}^{(1)} \quad \tilde{Q}_{31}^{(1)} \quad \tilde{Q}_{32}^{(1)} \quad \tilde{Q}_{33}^{(1)} \qquad\qquad\qquad\qquad (\pi = -) , \qquad (6)
\end{aligned}$$

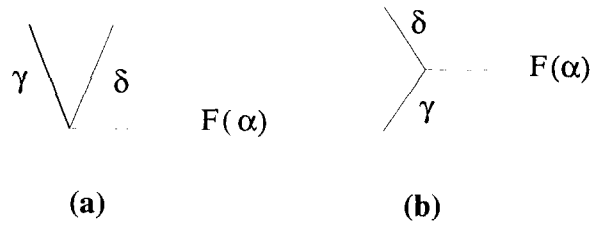

Fig. 1. Result of the action of a one-body operator in a quasiparticle basis. Graph (a) represents the creation of a two-quasiparticle excitation while graph (b) shows the scattering of a quasiparticle state into another state.

for the positive and negative signature, respectively. The symbol ˜ is used to denote that they are iso-stretched and doubly-stretched multipole operators [24–28].

The expressions of the operators in the quasiparticle representation are given by

$$\hat{R}^{(0)}_\rho = \sum_{\mu\bar{\nu}} \left[R^{(0)}_\rho(\mu\bar{\nu})\hat{A}^\dagger_{\mu\bar{\nu}} + \text{h.c.} \right] + \sum_{\mu\nu}{}'' R^{(0)}_\rho(\mu\nu)\hat{B}^\dagger_{\mu\nu} ,$$

$$\hat{R}^{(1)}_\rho = \sum_{\mu<\nu}{}'' \left[R^{(1)}_\rho(\mu\nu)\hat{A}^\dagger_{\mu\nu} + \text{h.c.} \right] + \sum_{\mu\bar{\nu}} \left[R^{(1)}_\rho(\mu\bar{\nu})\hat{B}^\dagger_{\mu\bar{\nu}} + \text{h.c.} \right] , \tag{7}$$

where the notation $\sum''_{\mu\nu}$ means that when a pair $(\mu\nu)$ is summed, its signature partner $(\bar{\mu}\bar{\nu})$ should also be summed, and use was made of the bilinear operators

$$\begin{cases} \hat{A}^\dagger_{\mu\nu} = \hat{a}^\dagger_\mu \hat{a}^\dagger_\nu , & \hat{A}^\dagger_{\mu\bar{\nu}} = \hat{a}^\dagger_\mu \hat{a}^\dagger_{\bar{\nu}} , \\ \hat{B}^\dagger_{\mu\nu} = \hat{a}^\dagger_\mu \hat{a}_\nu , & \hat{B}^\dagger_{\mu\bar{\nu}} = \hat{a}^\dagger_\mu \hat{a}_{\bar{\nu}} , \end{cases} \tag{8}$$

The operator $\hat{A}^\dagger$ creates two-quasiparticles while $\hat{B}^\dagger$ scatters one quasiparticle from one state to the other. These processes are shown in Fig. 1. In case (a) the generic single-particle operator $\hat{F}$ with signature α acts on the quasiparticle vacuum to give rise to a two-quasiparticle excitation. Because of the additivity of α, the signatures quantum number $\alpha_{\gamma(\delta)}$ of the quasiparticle states labelled by $\gamma(\delta)$ participating in the process depicted in Fig. 1a satisfy the relations

$$\begin{aligned} &\alpha = 0 \quad \alpha_\gamma + \alpha_\delta \equiv \alpha \implies \gamma \text{ and } \delta \text{ have opposite signature}, \\ &\alpha = 1 \quad \alpha_\gamma + \alpha_\delta \equiv \alpha \implies \gamma \text{ and } \delta \text{ have same signature}. \end{aligned} \tag{9}$$

Similarly, in the case of the process shown in Fig. 1b one finds

$$\begin{aligned} &\alpha = 0 \quad \alpha_\gamma \equiv \alpha_\delta + \alpha \implies \gamma \text{ and } \delta \text{ have same signature}, \\ &\alpha = 1 \quad \alpha_\gamma \equiv \alpha_\delta + \alpha \implies \gamma \text{ and } \delta \text{ have opposite signature}. \end{aligned} \tag{10}$$

Consequently, in the case where the operator has positive signature, the coefficients of the operators $\hat{A}^\dagger$ contain only combinations of states of opposite signature $(\mu\bar{\nu})$ while in the signature negative operators the states can be both $(\mu\nu)$ and $(\bar{\mu}\bar{\nu})$ (cf. Eq. (7)).

In this latter case a simple way to avoid double counting in carrying out the summations appearing in Eq. (7) is to impose the condition $\mu < \nu$ and $\bar{\mu} < \bar{\nu}$. Similar arguments to deduce the structure of the expansion coefficients of the term proportional to the $\hat{B}^{\dagger}$ operators can be carried out making now use of the conditions given in Eq. (10) for the quasiparticle quantum number of the states connected in the scattering process.

Summing up, the Hamiltonian describing the quasiparticle interaction responsible, among other things, for the presence of surface and pairing vibrations is

$$\hat{H} = \text{const.} + \sum_{\mu} E_{\mu} \hat{a}^{\dagger}_{\mu} \hat{a}_{\mu} + \sum_{\bar{\mu}} E_{\bar{\mu}} \hat{a}^{\dagger}_{\bar{\mu}} \hat{a}_{\bar{\mu}} + \hat{H}^{(0)}_{\text{int}} + \hat{H}^{(1)}_{\text{int}} . \tag{11}$$

In a rotating system, and even if the quasiparticles interact through separable forces, the RPA equations lead to a couple set of dispersion relations which, because of the large number of two-quasiparticle excitations (of the order of 10^4 for a typical medium-heavy nucleus), becomes numerically quite difficult to solve by searching for each single root. On the other hand, this search can be avoided making use of linear response theory. As an example, the RPA ground state correlation energy has been calculated by using this technique, without explicitly solving the RPA equations [12].

The definition of the RPA response function, which measures the change in the expectation value of the operator $\hat{R}^{(\alpha)}_{\rho}$ if we bring the system in an external field of type $\hat{R}^{(\alpha)}_{\rho}$, is

$$\mathcal{R} = (1 - S\chi)^{-1} S . \tag{12}$$

The so-called unperturbed response function S describes the response of the non-interacting system and is the quantity that contains explicitly the temperature dependence through Fermi factors. The expression of S is given in Appendix C while we refer the reader to Refs. [29,30] for details of the generalization of the RPA response function at finite temperature.

Both $\mathcal{R}$ and S are matrices with respect to the components of separable interactions. Using spectral representation, the linear response can be written as

$$\mathcal{R}^{(\alpha)}_{\rho\rho'}(\omega) = \sum_{q} \left(\frac{\mathcal{Q}^{(\alpha)}_{\rho}{}^{*}(q)\, \mathcal{Q}^{(\alpha)}_{\rho'}(q)}{\omega^{(\alpha)}_{q} - \omega} + \frac{\mathcal{Q}^{(\alpha)}_{\rho}(q)\, \mathcal{Q}^{(\alpha)}_{\rho'}{}^{*}(q)}{\omega^{(\alpha)}_{q} + \omega} \right) . \tag{13}$$

See Appendices C and D for the details of the derivation.

4. Self-energy in a rotating frame

As already mentioned, in a deformed rotating system the single-particle states carry signature and parity as good quantum numbers, quantum numbers which have to be conserved at each vertex coupling a particle to a vibration (cf. Fig. 1 and related discussion). Because of the lower symmetry of a deformed rotating nucleus as compared to the case of spherical nonrotating systems, the retarded self-energy needed in the

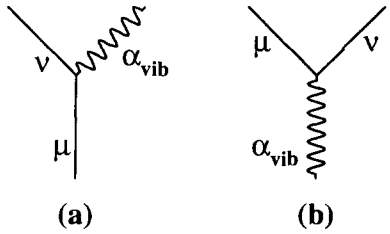

Fig. 2. Basic couplings between a quasiparticle and a collective mode which contribute to the self-energy.

calculation of e.g. the nucleon effective mass in the former case is expected to be more involved than that associated with the latter case [6].

4.1. Quasiparticle–vibration coupling

The starting point for defining the quasiparticle self-energy is the quasiparticle–vibration Hamiltonian describing the coupling between a quasiparticle and a surface or pairing vibrational mode which, in second quantization, are created by the operators $\hat{a}^{\dagger}_{\mu}$ and $\hat{X}^{\dagger(\alpha)}_{q}$, respectively, acting on the corresponding vacuum (BCS and RPA). The effective Hamiltonian describing the total system built up of single quasiparticles plus surface vibration can be written, in a rotating frame [31], as

$$\hat{H} = \sum_{\mu} E_{\mu}\hat{a}^{\dagger}_{\mu}\hat{a}_{\mu} + \sum_{\bar{\mu}} E_{\bar{\mu}}\hat{a}^{\dagger}_{\bar{\mu}}\hat{a}_{\bar{\mu}} + \sum_{\alpha}\sum_{q} \hbar\omega^{(\alpha)}_{q}\hat{X}^{\dagger(\alpha)}_{q}\hat{X}^{(\alpha)}_{q} + \sum_{\alpha}\hat{H}^{(\alpha)}_{\mathrm{coupl}}\,, \tag{14}$$

where $\hat{H}^{\alpha}_{\mathrm{coupl}}$ is the quasiparticle–vibration coupling Hamiltonian with signature quantum number α,

$$\hat{H}^{(\alpha)}_{\mathrm{coupl}} = -\sum_{\rho}\chi^{(\alpha)}_{\rho}\hat{h}^{(\alpha)}_{\rho}(\mathrm{vib})\hat{h}^{(\alpha)}_{\rho}(\mathrm{qp})\,. \tag{15}$$

The expressions of the vibration and quasiparticle Hamiltonian (both Hermitian operators) for different signature are given by

$$\hat{h}^{(\alpha)}_{\rho}(\mathrm{vib}) = \sum_{q}\left[\mathcal{Q}^{(\alpha)}_{\rho}(q)\hat{X}^{\dagger(\alpha)}_{q} + \mathrm{h.c.}\right], \tag{16}$$

and

$$\hat{h}^{(0)}_{\rho}(\mathrm{qp}) = \sum_{\mu\bar{\nu}}\left[R^{(0)}_{\rho}(\mu\bar{\nu})\hat{A}^{\dagger}_{\mu\bar{\nu}} + \mathrm{h.c.}\right] + \sum_{\mu\nu}{}'' R^{(0)}_{\rho}(\mu\nu)\hat{B}^{\dagger}_{\mu\nu}\,,$$

$$\hat{h}^{(1)}_{\rho}(\mathrm{qp}) = \sum_{\mu<\nu}{}'' \left[R^{(1)}_{\rho}(\mu\nu)\hat{A}^{\dagger}_{\mu\nu} + \mathrm{h.c.}\right] + \sum_{\mu\bar{\nu}}\left[R^{(1)}_{\rho}(\mu\bar{\nu})\hat{B}^{\dagger}_{\mu\bar{\nu}} + \mathrm{h.c.}\right]. \tag{17}$$

The main vertices of this quasiparticle–vibration coupling are depicted in Fig. 2. Graph (a) of Fig. 2 represents the scattering of a quasiparticle from a state μ to another state ν, through the creation of a vibrational mode. The process depicted in graph (b)

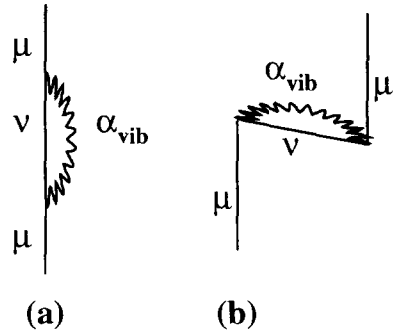

Fig. 3. The lowest second-order processes by which the quasiparticle motion is renormalized by the coupling to the collective mode.

of Fig. 2 corresponds to collective excitation with creation of a two quasiparticle-like excitation. These vertices are generated by application of different operators, that is, $\hat{B}^\dagger$ and $\hat{A}^\dagger$ in case (a) and (b), respectively. It is important to keep in mind this simple fact, as it has important consequences on the nature of the corresponding particle–vibration matrix elements associated with the process, differences which can be traced back to signature conservation (cf. Fig. 1 and related discussion). In order to keep track of these differences we shall, in what follows, distinguish between the self-energy diagram associated with vertices of "type" $\hat{B}^\dagger$ and of "type" $\hat{A}^\dagger$. This is done in Fig. 3, where graph (a) represents the self-energy correction due to the $\hat{B}^\dagger$ operator while graph (b) is the outcome of the $\hat{A}^\dagger$ operator. In the limit of a normal system, the second self-energy diagram will correspond to the situation in which the intermediate state is an hole state. Consequently, the associated energy denominator will be $-(E-(\varepsilon_\nu-\lambda)+\omega_q)$ while, in the superfluid case, the energy is $-(E+E_\nu+\omega_q)$. A representation of the variety of processes contributing to the quasiparticle self-energy which explicitly takes into account the conservation rules discussed above is shown in Fig. 4. From this figure it is seen that, for a particle of negative signature $\alpha = 1/2$ ($r = -i$), the intermediate state ν in diagram (a) of Fig. 3 has the same value of α if the collective mode has positive signature while it has opposite signature when the collective mode has $\alpha = 1$ (graphs (a) and (c) of Fig. 4). The situation is inverted when one considers diagram (b) of Fig. 3 where the operator $\hat{A}^\dagger$ is involved. The conservation of α produces graphs (b) and (d) of Fig. 4.

4.2. *Self-energy at finite temperature*

Taking into account the four possible diagrammatic contributions to the self-energy in a rotating frame, one can use finite temperature Green's function technique [21] to write the expression of the self-energy as

$$\Sigma_{\rm qp}(\xi, ip_n) = -\frac{1}{\beta}\sum_{i\omega_n}\sum_{\eta}\sum_{\alpha,\rho\rho'}\sum_{q}\frac{q^{(\alpha)}_{\rho\rho'}(\xi\eta)\,\mathcal{Q}^{(\alpha)*}_{\rho}(q)\,\mathcal{Q}^{(\alpha)}_{\rho'}(q)}{ip_n - s^{(\alpha)}_{\xi\eta}E_\eta - i\omega_n}$$

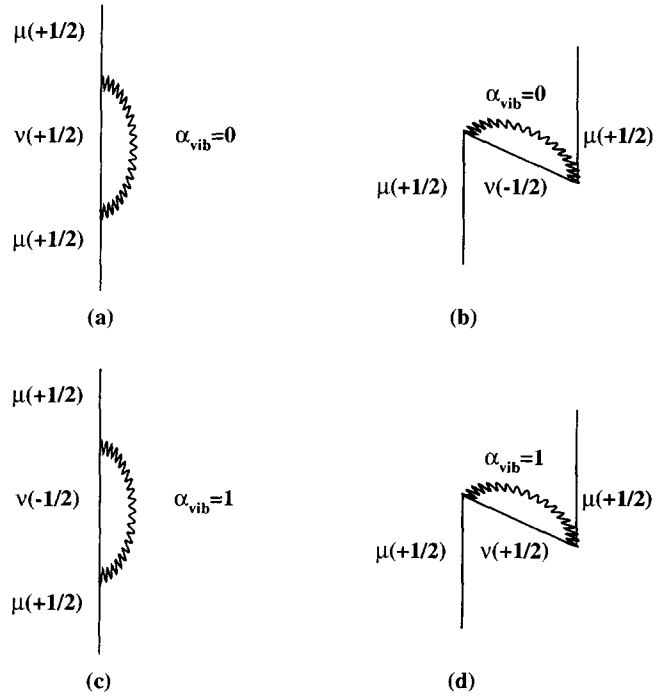

Fig. 4. Basic diagrams representing the self-energy of a quasiparticle state in a rotating frame. A wavy line stands for a collective vibration. Graphs (a) and (b) correspond to the case of vibrations with positive signature ($\alpha = 0$) while graphs (c) and (d) are associated with vibrations with negative signature ($\alpha = 1$).

$$\times \left[\frac{1}{i\omega_n - \omega_q^{(\alpha)}} - \frac{1}{i\omega_n + \omega_q^{(\alpha)}} \right], \tag{18}$$

where $i\omega_n = i2\pi n/\beta$ and $ip_n = i(2n+1)\pi/\beta$ ($n = 0, \pm 1, \pm 2, \ldots$) are the phonon and fermion frequencies and where

$$q^{(\alpha)}_{\rho\rho'}(\xi\eta) \equiv \chi^{(\alpha)}_{\rho} R^{(\alpha)}_{\rho}{}^{*}(\xi\eta) R^{(\alpha)}_{\rho'}(\xi\eta) \chi^{(\alpha)}_{\rho'}, \qquad s^{(\alpha)}_{\xi\eta} \equiv (-)^{\alpha+\alpha_\xi-\alpha_\eta}. \tag{19}$$

In the expression above an attempt to simplify the notation has been made. The states involved at the vertices of the self-energy diagrams (see Fig. 4) have been labelled by ξ for the initial state which can be either μ or $\bar{\mu}$, and the sum over the intermediate state η stands for states with both signatures ν and $\bar{\nu}$. Note that both the $A^\dagger$ and $B^\dagger$-type quasiparticle matrix elements in Eq. (17) appear as $R^{(\alpha)}_{\rho}(\xi\eta)$ and it is understood that $R^{(\alpha)}_{\rho}(\bar{\mu}\nu) = R^{(\alpha)}_{\rho}{}^{*}(\nu\bar{\mu})$. The signature of these states is indicated by $\alpha_{\xi(\eta)}$ while α is the vibrational signature index. The sign $s^{(\alpha)}_{\xi\eta}$ is necessary to distinguish the different behaviour of the energy denominators in case (b) and (d) of Fig. 4 as compared to cases (a) and (c) of the same figure. The notation $\Sigma_{\rm qp}(\xi, ip_n)$ stands for the self-energy correction of a quasiparticle state ξ with frequency ip_n.

Independently on its complexity, the self-energy in a rotating frame has the same structure as that corresponding to a nucleon moving in a spherical nonrotating nucleus (cf. Ref. [6]). Consequently, it is possible to write its retarded expression by using

the standard Matsubara's frequency summation method (see Ref. [6,21] for details) where one performs the sum over the phonon frequencies $i\omega_n$ followed by the analytical continuation $ip_n \to E + iI$, namely

$$\Sigma_{\rm ret}(\xi, E+iI) = \sum_{\eta} \sum_{\alpha,\rho\rho'} \sum_{q} q^{(\alpha)}_{\rho\rho'}(\xi\eta)\, \mathcal{Q}^{(\alpha)*}_{\rho}(q)\, \mathcal{Q}^{(\alpha)}_{\rho'}(q) \times \left[\frac{1 + n_B(\omega^{(\alpha)}_q) - n_F(s^{(\alpha)}_{\xi\eta} E_\eta)}{E + iI - s^{(\alpha)}_{\xi\eta} E_\eta - \omega^{(\alpha)}_q} + \frac{n_B(\omega^{(\alpha)}_q) + n_F(s^{(\alpha)}_{\xi\eta} E_\eta)}{E + iI - s^{(\alpha)}_{\xi\eta} E_\eta + \omega^{(\alpha)}_q} \right], \quad (20)$$

where the Bose and Fermi occupation factors are defined,

$$n_B(E) = (e^{\beta E} - 1)^{-1}, \qquad n_F(E) = (e^{\beta E} + 1)^{-1}. \quad (21)$$

As already pointed out, the RPA equations are too involved to be solved by finding each individual root, and consequently Eq. (20) cannot be used directly, as it was done in [6–8]. We have to rewrite the term containing the sum over the phonon indices q in terms of the response function, which contains all the informations about the collective vibrations in an implicit form. To be able to use the linear response function, we need to proceed in a different way compared to the conventional Matsubara's technique. In fact, it is convenient to modify the original expression of the self-energy defined in Eq. (18) introducing at this level the analytical continuation $ip_n \to E + iI$, followed by the summation over the phonon frequencies. This is simply an inversion in the order of the steps which are carried out in the standard Matsubara's method but, as we shall see below, it leads to the possibility of intoduce the linear response function in the self-energy. In fact, the analytical continuation modifies the poles of the fermion Green's function,

$$\frac{1}{ip_n - s^{(\alpha)}_{\xi\eta} E_\eta - i\omega_n}, \qquad ip_n \to E + iI, \quad (22)$$

which, after performing the phonon frequency summation, modifies the standard expression (see Eq. (20)) of the self-energy contribution. The details of this method are quite lengthly. We provide the main steps in Appendix E and F, while for further details we refer the reader to [32].

In any case, the retarded self-energy in a rotating frame can be expressed as

$$\Sigma_{\rm ret}(\xi, E+iI) = \sum_{\eta} \sum_{\alpha,\rho\rho'} q^{(\alpha)}_{\rho\rho'}(\xi\eta) \left[\frac{1}{2\beta} \sum_{i\omega_n} \frac{\mathcal{R}^{(\alpha)}_{\rho\rho'}(i\omega_n) + \mathcal{R}^{(\alpha)}_{\rho'\rho}(i\omega_n)}{E + iI - s^{(\alpha)}_{\xi\eta} E_\eta - i\omega_n} + \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \left[1 + n_B(i\omega) - n_F(s^{(\alpha)}_{\xi\eta} E_\eta) \right] \frac{\mathcal{R}^{(\alpha)}_{\rho\rho'}(i\omega) - \mathcal{R}^{(\alpha)}_{\rho'\rho}(i\omega)}{E + iI - s^{(\alpha)}_{\xi\eta} E_\eta - i\omega} - \left[1 + n_B(E + iI - s^{(\alpha)}_{\xi\eta} E_\eta) - n_F(s^{(\alpha)}_{\xi\eta} E_\eta) \right] \times \begin{cases} \mathcal{R}^{(\alpha)}_{\rho\rho'}(E + iI - s^{(\alpha)}_{\xi\eta} E_\eta), & \text{if } E > s^{(\alpha)}_{\xi\eta} E_\eta \\ \mathcal{R}^{(\alpha)}_{\rho'\rho}(E + iI - s^{(\alpha)}_{\xi\eta} E_\eta), & \text{if } E < s^{(\alpha)}_{\xi\eta} E_\eta \end{cases} \right], \quad (23)$$

in terms of the response function of the system.

4.3. Self-energy at zero temperature

As discussed in [21], the limiting case of zero temperature can be obtained from the finite temperature expression substituting the sum over boson frequencies ($i\omega_n$) with a proper integration on the complex plane. Proper limits in Bose and Fermi occupation factors should also be taken into account,

$$n_B(z) \to \begin{cases} 0 & \text{if } \mathrm{Re}\, z > 0, \\ -\frac{1}{2} & \text{if } \mathrm{Re}\, z = 0, \\ -1 & \text{if } \mathrm{Re}\, z < 0, \end{cases} \quad \text{and} \quad n_F(s^{(\alpha)}_{\xi\eta} E_\eta) \to \frac{1 - s^{(\alpha)}_{\xi\eta}}{2}. \tag{24}$$

Because quasiparticle energies are positive quantities, the limit of Fermi factor depends on the sign $s^{(\alpha)}_{\xi\eta}$. One can now write the final expression

$$\begin{aligned} \Sigma_{\mathrm{ret}}(\xi, E + iI) = \sum_{\eta} \sum_{\alpha,\rho\rho'} q^{(\alpha)}_{\rho\rho'}(\xi\eta) \Bigg[& \frac{1}{2} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\mathcal{R}^{(\alpha)}_{\rho\rho'}(i\omega) + \mathcal{R}^{(\alpha)}_{\rho'\rho}(i\omega)}{E + iI - s^{(\alpha)}_{\xi\eta} E_\eta - i\omega} \\ & + \frac{s^{(\alpha)}_{\xi\eta}}{2} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\mathcal{R}^{(\alpha)}_{\rho\rho'}(i\omega) - \mathcal{R}^{(\alpha)}_{\rho'\rho}(i\omega)}{E + iI - s^{(\alpha)}_{\xi\eta} E_\eta - i\omega} \\ & - \begin{cases} \dfrac{s^{(\alpha)}_{\xi\eta} + 1}{2} \mathcal{R}^{(\alpha)}_{\rho\rho'}(E + iI - s^{(\alpha)}_{\xi\eta} E_\eta), & \text{if } E > s^{(\alpha)}_{\xi\eta} E_\eta \\ \dfrac{s^{(\alpha)}_{\xi\eta} - 1}{2} \mathcal{R}^{(\alpha)}_{\rho'\rho}(E + iI - s^{(\alpha)}_{\xi\eta} E_\eta), & \text{if } E < s^{(\alpha)}_{\xi\eta} E_\eta \end{cases} \Bigg] \end{aligned} \tag{25}$$

of the retarded self-energy at zero temperature in terms of the RPA response functions.

4.4. Effective mass

Once the retarded self-energy is obtained the renormalized energy of the quasiparticle can be calculated [2,33] according to

$$\tilde{E}_\xi = E_\xi + \mathrm{Re}\, \Sigma_{\mathrm{ret}}(\xi, \tilde{E}_\xi + iI). \tag{26}$$

The results are found to be quite stable with respect to variations in the energy averaging parameter I [2]. The effective mass (ω-mass), for each quasiparticle state, can be calculated as the inverse of the residue of the quasiparticle Green function and given by

$$\frac{m_\omega(\xi)}{m} = 1 - \left. \frac{d\, \mathrm{Re}\, \Sigma_{\mathrm{ret}}(\xi, E + iI)}{dE} \right|_{E=\tilde{E}_\xi}. \tag{27}$$

Making use of this state dependent relation, the ω-mass is defined in terms of an average over levels lying near the Fermi energy.

5. Conclusions

The particle–vibration coupling is the single, most important mechanism at the basis of the dressing of quasiparticles, as testified by the structure of the relations describing effective masses and charges, induced interactions, etc.

In the present paper we have generalized the finite temperature particle–vibration coupling formalism developed to deal with spherical nuclei, to the case of deformed, rotating nuclei making use of linear response theory. This, in keeping with the fact that in lower symmetry systems (as compared to spherical situations), the vibrational spectrum is very rich and complex. Once the particle–vibration vertex has been expressed in terms of response functions, any quantity resulting from the interweaving of quasiparticle and vibrational mode can be worked out. The method has been exemplified by working out a closed, compact expression of the quasiparticle self-energy, needed, for example, in connection with the calculation of the ω-mass.

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Appendix A. Mean-field in the rotating frame and signature basis

In a rotating frame a system is invariant with respect to rotations by π about an axis perpendicular to the symmetry axis. This operation can be carried out through the operator $\hat{R}_x$ defined as $\hat{R}_x = e^{-i\pi \hat{J}_x}$. Denoting by r the eigenvalues of $\hat{R}_x$ and introducing the notation $r = e^{-i\pi\alpha}$, one finds for systems with an even number of nucleons (integer total spin I),

$$\begin{aligned} &r = +1 \quad (\alpha = 0), \quad I = 0, 2, 4, \ldots, \\ &r = -1 \quad (\alpha = 1), \quad I = 1, 3, 5, \ldots, \end{aligned} \tag{A.1}$$

while for systems with odd particle number we have (half-integer total spin I),

$$\begin{aligned} &r = -i \quad (\alpha = +\tfrac{1}{2}), \quad I = \tfrac{1}{2}, \tfrac{5}{2}, \tfrac{9}{2}, \ldots, \\ &r = +i \quad (\alpha = -\tfrac{1}{2}), \quad I = \tfrac{3}{2}, \tfrac{7}{2}, \tfrac{11}{2}, \ldots. \end{aligned} \tag{A.2}$$

The eigenvalues r associated with the rotation operator $\hat{R}_x$ can be used to classify the single-particle states. However, due to its additive nature, it is more convenient define α as *signature* quantum number.

In the standard basis where m_i is the projection of the angular momentum on the symmetry axis z, the operator $\hat{J}_x$ is not block diagonal with respect to the quantum number m_i. This property comes from the breaking of the time reversal invariance. We restrict the members of the standard basis to states that satisfy the condition $m_i > 0$ so that the dimension of the equations will be reduced by half [35]. Moreover, the chosen basis is symmetrized with respect to a rotation of π around the rotational axis x, namely we introduce the signature α as good quantum number to label the quasiparticle states.

This new basis can be defined in terms of the original nucleon creation and annihilation operators $\hat{c}_i^\dagger, \hat{c}_i$, as

$$\begin{pmatrix} \hat{d}_i^\dagger \\ \hat{d}_{\bar{i}}^\dagger \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & (-)^{m-1/2} \\ (-)^{m+1/2} & 1 \end{pmatrix} \begin{pmatrix} \hat{c}_i^\dagger \\ \hat{c}_{\bar{i}}^\dagger \end{pmatrix}. \tag{A.3}$$

The notation $\tilde{i}$ indicates the time-reversal state to i while $(\bar{i},i)$ represents, as we will see below (A.5), a signature pair. We can now introduce the appropriate Bogoliubov–Valatin transformation that diagonalize the HFB Hamiltonian (1),

$$\begin{aligned} \hat{a}_\mu^\dagger &= \sum_i{}' \left(U_{\mu i} \hat{d}_i^\dagger + V_{\mu i} \hat{d}_{\bar{i}} \right), \\ \hat{a}_{\bar{\mu}}^\dagger &= \sum_i{}' \left(\bar{U}_{\mu i} \hat{d}_{\bar{i}}^\dagger + \bar{V}_{\mu i} \hat{d}_i \right), \end{aligned} \tag{A.4}$$

where $U, V, \bar{U}$ and $\bar{V}$ are real coefficients. The notation $\sum_i'$ is used to specify that we are summing over the states of the new basis (A.3). The quasiparticle operators $(\hat{a}_\mu^\dagger, \hat{a}_{\bar{\mu}}^\dagger)$ satisfy the property

$$e^{-i\pi \hat{J}_x} \begin{pmatrix} a_\mu^\dagger \\ a_{\bar{\mu}}^\dagger \end{pmatrix} e^{i\pi \hat{J}_x} = \mp i \begin{pmatrix} a_\mu^\dagger \\ a_{\bar{\mu}}^\dagger \end{pmatrix}. \tag{A.5}$$

This implies that the quasiparticle state μ has negative signature $\alpha = +1/2$ $(r = -i)$, while $\bar{\mu}$ is the state with positive signature $\alpha = -1/2$ $(r = +i)$.

The quasiparticle energies E_μ and $E_{\bar{\mu}}$ are eigenvalues and the coefficients $U, V, \bar{U}, \bar{V}$ eigenvectors of the matrix

$$\mathcal{M} = \begin{pmatrix} (\varepsilon - \lambda I - \omega_{\text{rot}} \hat{J}_x) & \Delta \\ \Delta & -(\varepsilon - \lambda I + \omega_{\text{rot}} \hat{J}_x) \end{pmatrix}, \tag{A.6}$$

which is derived from the mean-field Hamiltonian, Eq. (1), that is,

$$\mathcal{M} \begin{pmatrix} U_\mu \\ V_\mu \end{pmatrix} = E_\mu \begin{pmatrix} U_\mu \\ V_\mu \end{pmatrix}, \qquad \mathcal{M} \begin{pmatrix} \bar{V}_{\bar{\mu}} \\ \bar{U}_{\bar{\mu}} \end{pmatrix} = -E_{\bar{\mu}} \begin{pmatrix} \bar{V}\bar{\mu} \\ \bar{U}\bar{\mu} \end{pmatrix}. \tag{A.7}$$

Appendix B. Residual interaction classified by signature

We consider, first of all, the residual pairing interaction associated with the pairing potential $-\Delta(\hat{P}^\dagger + \hat{P})$, where $\hat{P}^\dagger$ is the monopole pair operator. It can be written as

$$\hat{H}_{\rm P} = -\sum_\tau G_\tau \tilde{P}^\dagger_\tau \tilde{P}_\tau\,, \qquad \tilde{P}^\dagger_\tau = \hat{P}^\dagger_\tau - \langle \hat{P}^\dagger_\tau \rangle\,, \tag{B.1}$$

where the index τ is used to distinguish between proton and neutron single-particle states. The expectation value is taken in the quasiparticle vacuum. This expectation value is related to the pairing deformation Δ through the self-consistency condition $\Delta_\tau = G_\tau \langle \hat{P}^\dagger_\tau \rangle$.

The pairing residual interaction can be expressed with the help of the operators

$$\tilde{P}_{\tau+} = \hat{P}^\dagger_\tau + \hat{P}_\tau\,, \qquad \tilde{P}_{\tau-} = i(\hat{P}^\dagger_\tau - \hat{P}_\tau)\,, \tag{B.2}$$

which satisfy the hermiticity condition $\tilde{P}^\dagger_{\tau\pm} = \tilde{P}_{\tau\pm}$. These operators carry positive signature because

$$e^{-i\pi \hat{J}_x} \tilde{P}_{\tau\pm} e^{i\pi \hat{J}_x} = +\tilde{P}_{\tau\pm}\,. \tag{B.3}$$

Making use of the definitions of Eq. (B.2), the pairing residual interaction can thus be written as

$$\hat{H}_P = -\sum_\tau \sum_{\rho=\pm} G_{\tau\rho} \tilde{P}^\dagger_{\tau\rho} \tilde{P}_{\tau\rho}\,. \tag{B.4}$$

For the multipole–multipole interaction associated with deformation the situation is slightly more complicated. In fact, the interaction is defined in terms of the multipole operators as

$$\hat{H}_L = -\tfrac{1}{2}\chi_L \hat{Q}^\dagger_{LK} \hat{Q}_{LK}\,, \qquad \hat{Q}_{LK} = \sum_{a=1}^{A} (r^L Y_{LK})_a\,, \tag{B.5}$$

where the spherical harmonic functions Y_{LK} are defined with respect to the symmetry axis z. We shall only consider the quadrupole ($L = 2$) and octupole ($L = 3$) terms. Note that we should modify the operators in the residual interaction in such a way to include the effect of deformation. This is nothing but the use of the doubly-stretched [28] and iso-stretched [24] multipole operators. We have furthermore to remember that the K-quantum number, that is, the projection of the total angular momentum along the symmetry axis, is no longer a conserved quantity at finite rotational frequency. As a consequence, it is convenient to deal with operators defined in terms of the quantum numbers which are conserved in a rotating frame, namely the parity π and the signature α (see Appendix A). For this purpose, one can use the following definition of the multipole operators [36]:

$$\tilde{Q}^{(\alpha)}_{LK} = \frac{i^{L+\alpha+K}}{\sqrt{2(1+\delta_{K0})}} \left(r^L Y_{LK} + (-)^{L+\alpha} r^L Y_{L-K}\right), \tag{B.6}$$

where $K \geq 0$ and α can assume the values 0 and 1. The multipole operators with $\alpha = 0$ are called signature positive operators while $\alpha = 1$ corresponds to signature negative operators. This difference arizes from the fact that

$$e^{-i\pi\hat{J}_x}\tilde{Q}^{(0)}_{LK}e^{i\pi\hat{J}_x} = +\tilde{Q}^{(0)}_{LK}, \qquad e^{-i\pi\hat{J}_x}\tilde{Q}^{(1)}_{LK}e^{i\pi\hat{J}_x} = -\tilde{Q}^{(1)}_{LK}. \tag{B.7}$$

The factor $i^{L+\alpha+K}$ is introduced to ensure hermiticity of the multipole operators, that is $\tilde{Q}^{\dagger(\alpha)}_{LK} = \tilde{Q}^{(\alpha)}_{LK}$. Note that the $K = 0$ quadrupole (octupole) operator $\tilde{Q}_{20}$ ($\tilde{Q}_{30}$) has a unique signature $\alpha = 0$ ($\alpha = 1$), which corresponds to the fact that $K = 0$ bands have no signature partners. Using the usual phase convention of the single-particle orbits, the matrix elements of all the operators in Eqs. (B.2) and (B.6) can be chosen to be either real or pure imaginary. Taking into account these considerations, we then obtain the residual interaction of the form given in Eq. (4) in the main text.

Appendix C. Coupled RPA and response functions

The excitation operators of the RPA normal modes $\hat{X}^{\dagger(\alpha)}_q$ are constructed in terms of the quasiparticle operators as

$$\hat{X}^{\dagger(0)}_q = \sum_{\mu\bar{\nu}} \left\{ \psi^{(0)}_q(\mu\bar{\nu})\hat{A}^{\dagger}_{\mu\bar{\nu}} + \varphi^{(0)}_q(\mu\bar{\nu})\hat{A}_{\mu\bar{\nu}} \right\}, \tag{C.1}$$

for positive-type operators and

$$\hat{X}^{\dagger(1)}_q = \sum_{\mu<\nu}{}'' \left\{ \psi^{(1)}_q(\mu\nu)\hat{A}^{\dagger}_{\mu\nu} + \varphi^{(1)}_q(\mu\nu)\hat{A}_{\mu\nu} \right\}, \tag{C.2}$$

for negative-type operators. The index q specifies the RPA eigenmodes, and $\psi^{(\alpha)}_q(\xi\eta)$ and $\varphi^{(\alpha)}_q(\xi\eta)$ are the forwarding and backwarding RPA amplitudes (here we use ξ and η to denote the quasiparticle indices when the signature quantum number, $\alpha = \pm 1/2$, is not specified).

The equation of motion and the normalization condition in the RPA theory are given by

$$[\hat{H}, \hat{X}^{\dagger(\alpha)}_q]_{\mathrm{RPA}} = \omega^{(\alpha)}_q \hat{X}^{\dagger(\alpha)}_q, \qquad [\hat{X}^{(\alpha)}_q, \hat{X}^{\dagger(\alpha)}_{q'}]_{\mathrm{RPA}} = \delta_{qq'}, \tag{C.3}$$

where the notation $[\ \ ]_{\mathrm{RPA}}$ means that we neglect the terms higher then the RPA order in the commutator.

Let us denote the transition matrix elements between the RPA excited states q and the yrast state as

$$\begin{aligned} \mathcal{Q}^{(\alpha)}_{\rho}(q) &= \langle q|\hat{R}^{(\alpha)}_{\rho}|\omega_{\mathrm{rot}}\rangle = [\hat{X}^{(\alpha)}_q, \hat{R}^{(\alpha)}_{\rho}]_{\mathrm{RPA}}, \\ \mathcal{Q}^{(\alpha)*}_{\rho}(q) &= \langle \omega_{\mathrm{rot}}|\hat{R}^{(\alpha)}_{\rho}|q\rangle = [\hat{R}^{(\alpha)}_{\rho}, \hat{X}^{\dagger(\alpha)}_q]_{\mathrm{RPA}}. \end{aligned} \tag{C.4}$$

Then, the equation of motion (C.3) is equivalent to the linear homogenous equation

$$\mathcal{Q}_{\rho}^{(\alpha)*}(q) = \sum_{\rho'} S_{\rho\rho'}^{(\alpha)}(\omega)\,\mathcal{Q}_{\rho'}^{(\alpha)*}(q)\,, \tag{C.5}$$

where the quantities $S_{\rho\rho'}^{(\alpha)}(\omega)$ are the so-called unperturbed response functions necessary to calculate the vibrational modes. In fact, RPA eigenenergies $\omega_q^{(\alpha)}$ are obtained by solving the equation

$$\det\left(\chi_{\rho}^{(\alpha)} S_{\rho\rho'}^{(\alpha)}(\omega) - \delta_{\rho\rho'}\right) = 0\,. \tag{C.6}$$

This relation corresponds to the condition that (C.5) has a nontrivial solution. This is a determinant equation of dimension 7×7 in the $\alpha = 0$ sector and 2×2 in $\alpha = 1$ for the pairing plus quadrupole interaction channel, and of dimension 3×3 in $\alpha = 0$ and 4×4 in $\alpha = 1$ for the octupole interaction channel, respectively, as one can see from the definitions given in Eqs. (5) and (6) of the residual interactions. Each RPA eigenstate is characterized by the corresponding forwarding and backwarding amplitudes,

$$\begin{aligned}
\psi_q^{(\alpha)}(\xi\eta) &= +\frac{\sum_{\rho}\chi_{\rho}^{(\alpha)}\mathcal{Q}_{\rho}^{(\alpha)*}(q)\,R_{\rho}^{(\alpha)}(\xi\eta)}{E_{\xi}+E_{\eta}-\omega_q^{(\alpha)}}\times N_q\,,\\
\varphi_q^{(\alpha)}(\xi\eta) &= -\frac{\sum_{\rho}\chi_{\rho}^{(\alpha)}\mathcal{Q}_{\rho}^{(\alpha)*}(q)\,R_{\rho}^{(\alpha)*}(\xi\eta)}{E_{\xi}+E_{\eta}+\omega_q^{(\alpha)}}\times N_q\,,
\end{aligned} \tag{C.7}$$

where N_q is the normalization constant obtained from the relation

$$\sum_{q}\left\{\psi_q^{*(\alpha)}(\xi\eta)\psi_q^{(\alpha)}(\xi\eta) - \varphi_q^{*(\alpha)}(\xi\eta)\varphi_q^{(\alpha)}(\xi\eta)\right\} = 1\,. \tag{C.8}$$

Here we use the notation $(\xi,\eta) = (\mu,\bar{\nu})$ for $\alpha = 0$ and $(\xi,\eta) = (\mu,\nu)$, $(\bar{\mu},\bar{\nu})$ for $\alpha = 1$.

The transition matrix elements between the RPA excited states q and the yrast state for any of the residual interaction can be expressed in terms of the amplitudes $\psi_q^{(\alpha)}(\xi\eta)$ and $\varphi_q^{(\alpha)}(\xi\eta)$,

$$\mathcal{Q}_{\rho}^{(\alpha)*}(q) = \sum_{\xi\eta}\left[R_{\rho}^{(\alpha)*}(\xi\eta)\psi_q^{(\alpha)}(\xi\eta) - R_{\rho}^{(\alpha)}(\xi\eta)\varphi_q^{(\alpha)}(\xi\eta)\right]. \tag{C.9}$$

The expression for the unperturbed response functions for positive and negative signature is given by

$$\begin{aligned}
S_{\rho\rho'}^{(0)}(\omega) &= \sum_{\mu\bar{\nu}}\left(\frac{R_{\rho}^{(0)*}(\mu\bar{\nu})R_{\rho'}^{(0)}(\mu\bar{\nu})}{E_{\mu\bar{\nu}}-\omega} + \frac{R_{\rho}^{(0)}(\mu\bar{\nu})R_{\rho'}^{(0)*}(\mu\bar{\nu})}{E_{\mu\bar{\nu}}+\omega}\right),\\
S_{\rho\rho'}^{(1)}(\omega) &= \sum_{\mu\nu}{}'\left(\frac{R_{\rho}^{(1)*}(\mu\nu)R_{\rho'}^{(1)}(\mu\nu)}{E_{\mu\nu}-\omega} + \frac{R_{\rho}^{(1)}(\mu\nu)R_{\rho'}^{(1)*}(\mu\nu)}{E_{\mu\nu}+\omega}\right),
\end{aligned} \tag{C.10}$$

where the quantities $E_{\mu\bar{\nu}} = E_{\mu} + E_{\bar{\nu}}$ and $E_{\mu\nu} = E_{\mu} + E_{\nu}$ are the two-quasiparticle excitations in the positive and negative sector, respectively. In the case of a nonsuperconducting

phase these energies correspond to the usual particle–hole excitations. At finite temperature the quasiparticle matrix elements, $R_\rho^{(\alpha)}(\xi\eta)$, include the Fermi occupation factors in a usual way [30].

Appendix D. Linear response theory

It is convenient to have the explicit expression for the response function, namely its spectral representation (see [37]). For this purpose we need to calculate the resolvent operator

$$\hat{G}(\omega) = \left[\hat{H} - \omega\right]^{-1}. \tag{D.1}$$

The matrix $G(\omega)$ corresponding to the above resolvent is easily obtained once the RPA eigenvalue problem is solved. The RPA spectrum consists of complex energies. However, one can show that only the real ones correspond to physical solutions, while the complex ones are sign of the instability of the system. For nonzero real solutions, the eigenvalues and eigenvectors of the RPA can be written as $(\omega_q^{(\alpha)}, X_q^{(\alpha)}), (-\omega_q^{(\alpha)}, \bar{X}_q^{(\alpha)})$, where the eigenvectors are defined by

$$\begin{aligned} X_q^{(\alpha)} &= \begin{pmatrix} \psi_q^{(\alpha)}(\xi\eta) \\ \varphi_q^{(\alpha)}(\xi\eta) \end{pmatrix} \quad \leftrightarrow \quad \hat{X}_q^{\dagger(\alpha)} = \sum_{\xi<\eta} \left[\psi_q^{(\alpha)}(\xi\eta)\hat{A}_{\xi\eta}^\dagger + \varphi_q^{(\alpha)}(\xi\eta)\hat{A}_{\xi\eta}\right], \\ \bar{X}_q^{(\alpha)} &= \begin{pmatrix} \varphi_q^{(\alpha)*}(\xi\eta) \\ \psi_q^{(\alpha)*}(\xi\eta) \end{pmatrix} \quad \leftrightarrow \quad \hat{X}_q^{(\alpha)} = \sum_{\xi<\eta} \left[\varphi_q^{(\alpha)*}(\xi\eta)\hat{A}_{\xi\eta}^\dagger + \psi_q^{(\alpha)*}(\xi\eta)\hat{A}_{\xi\eta}\right]. \end{aligned} \tag{D.2}$$

The spectral representation of the resolvent matrix expressed in terms of these eigenvectors is written as

$$G^{(\alpha)}(\omega) = \sum_q \left[\frac{X_q^{(\alpha)} * X_q^{\dagger(\alpha)}}{\omega_q^{(\alpha)} - \omega} + \frac{\bar{X}_q^{(\alpha)} * \bar{X}_q^{\dagger(\alpha)}}{\omega_q^{(\alpha)} + \omega}\right], \tag{D.3}$$

where the notation $X*Y^\dagger$ represents a matrix constructed from the product of two vectors X and $Y^\dagger$ (notice that this is valid only when we do not have zero-energy solutions, that is spurious states).

The resolvent matrix can be used to evaluate the spectral representation of the RPA response function for two generic one-body operators, $\hat{A}$ and $\hat{B}$, in the following way:

$$\mathcal{R}_{AB}^{(\alpha)}(\omega) \equiv A^\dagger G^{(\alpha)}(\omega) B, \tag{D.4}$$

where A and B are column vectors representing the RPA part (see Eq. (D.2)) of the operators. This means that the numerators of the response function will contains terms such as $A^\dagger X_q^{(\alpha)} * X_q^{(\alpha)\dagger} B$ that, in the framework of the RPA response, can be written as

$$A^\dagger X_q^{(\alpha)} * X_q^{(\alpha)\dagger} B = [\hat{A}^\dagger, \hat{X}_q^{(\alpha)\dagger}]_{\mathrm{RPA}} [\hat{X}_q^{(\alpha)}, \hat{B}]_{\mathrm{RPA}} = \mathcal{Q}_A^{(\alpha)*}(q)\mathcal{Q}_B^{(\alpha)}(q), \tag{D.5}$$

where is used the definition of the transition matrix elements between excited and ground state (see Eq. (C.4)).

Introducing this last equality in the spectral representation of the RPA response functions. Especially what is important in our formulation is a matrix of the RPA response functions, in which the raw and the column are specified by the hermite operators which consist of the coupled-separable residual interactions in Eq. (4),

$$\mathcal{R}^{(\alpha)}_{\rho\rho'}(\omega) \equiv \mathcal{R}^{(\alpha)}_{R^{(\alpha)}_{\rho} R^{(\alpha)}_{\rho'}}(\omega) = \sum_{q} \left(\frac{\mathcal{Q}^{(\alpha)*}_{\rho}(q)\mathcal{Q}^{(\alpha)}_{\rho'}(q)}{\omega^{(\alpha)}_{q} - \omega} + \frac{\mathcal{Q}^{(\alpha)}_{\rho}(q)\mathcal{Q}^{(\alpha)*}_{\rho'}(q)}{\omega^{(\alpha)}_{q} + \omega} \right). \quad \text{(D.6)}$$

The RPA equation is equivalent to the Dyson equation in terms of the resolvent

$$G(\omega) = [1 + G_0(\omega)V]^{-1} G_0(\omega), \quad \text{(D.7)}$$

where $G_0(\omega)$ is the unperturbed one corresponding to the mean-field Hamiltonian, Eq. (3), and the coupled-separable type residual interaction, Eq. (4), is expressed as a matrix in the RPA space given by

$$V = -\sum_{\rho} \chi_{\rho} R^{(\alpha)}_{\rho} * R^{(\alpha)\dagger}_{\rho'}. \quad \text{(D.8)}$$

Thus, the RPA response functions in Eq. (D.6) can be directly calculated by

$$\mathcal{R} = (1 - S\chi)^{-1} S, \quad \text{(D.9)}$$

where $\mathcal{R}$, S and χ (here $(\chi)_{\rho\rho'} = \chi_{\rho}\delta_{\rho\rho'}$), in the actual case of coupled RPA, are $N \times N$ matrices, where N is the total number of the residual interactions in a specific sector of signature and parity. The matrix S is the unperturbed response function defined in terms of two-quasiparticle excitations given by Eq. (C.10).

Appendix E. New frequency summation

We start carrying out the standard boson frequency summation of Eq. (18) as explained in the textbook [21]. The result is

$$\Sigma_{\mathrm{qp}}(\xi, ip_n) = \sum_{\eta} \sum_{\alpha,\rho\rho'} \sum_{q} q^{(\alpha)}_{\rho\rho'}(\xi\eta)\mathcal{Q}^{(\alpha)*}_{\rho}(q)\mathcal{Q}^{(\alpha)}_{\rho'}(q) \times \left[\frac{1 + n_B(\omega^{(\alpha)}_{q}) - n_F(s^{(\alpha)}_{\xi\eta}E_{\eta})}{ip_n - s^{(\alpha)}_{\xi\eta}E_{\eta} - \omega^{(\alpha)}_{q}} + \frac{n_B(\omega^{(\alpha)}_{q}) + n_F(s^{(\alpha)}_{\xi\eta}E_{\eta})}{ip_n - s^{(\alpha)}_{\xi\eta}E_{\eta} + \omega^{(\alpha)}_{q}} \right], \quad \text{(E.1)}$$

where the relation $n_B(ip_n - E) = n_F(E) - 1$ was used (as a consequence of the definition $ip_n = i(2n+1)\pi/\beta$). The analytic continuation, $ip_n \to E + iI$, of this equation leads to the retarded self-energy (Eq. (20)). The obstacle that prevents us to introduce the linear response function directly in Eq. (20) is the presence of the Bose and Fermi factors in the numerators in the square backet, as they do not appear in the RPA response functions (cf. Eq. (D.6)). To avoid this problem one needs to find a different method for carrying out the summation over the phonon frequencies, when one has first made

the analytical continuation, that is when one has made the substitution $ip_n \to E + iI$ which leads to

$$\Sigma_{\rm qp}(\xi, E+iI) = -\frac{1}{\beta}\sum_{i\omega_n}\sum_{\eta}\sum_{\alpha,\rho\rho'}\sum_{q}\frac{q^{(\alpha)}_{\rho\rho'}(\xi\eta)\,\mathcal{Q}^{(\alpha)*}_{\rho}(q)\,\mathcal{Q}^{(\alpha)}_{\rho'}(q)}{E+iI-s^{(\alpha)}_{\xi\eta}E_\eta - i\omega_n} \times\left[\frac{1}{i\omega_n-\omega^{(\alpha)}_q}-\frac{1}{i\omega_n+\omega^{(\alpha)}_q}\right]. \tag{E.2}$$

Carrying out the sum over the phonon frequencies at this stage, one obtains

$$\Sigma_{\rm qp}(\xi, E+iI) = \sum_{\eta}\sum_{\alpha,\rho\rho'}\sum_{q} q^{(\alpha)}_{\rho\rho'}(\xi\eta)\,\mathcal{Q}^{(\alpha)*}_{\rho}(q)\,\mathcal{Q}^{(\alpha)}_{\rho'}(q) \times\left[\frac{n_B(\omega^{(\alpha)}_q)-n_B(E+iI-s^{(\alpha)}_{\xi\eta}E_\eta)}{E+iI-s^{(\alpha)}_{\xi\eta}E_\eta-\omega^{(\alpha)}_q} +\frac{1+n_B(\omega^{(\alpha)}_q)+n_B(E+iI-s^{(\alpha)}_{\xi\eta}E_\eta)}{E+iI-s^{(\alpha)}_{\xi\eta}E_\eta+\omega^{(\alpha)}_q}\right]. \tag{E.3}$$

The main point to be understood here is that the two results, Eqs. (E.1) and (E.3), are different because of the inequality $n_B(E+iI-E') \neq n_F(E')-1$, which comes from the modification of the Fermion Green's function when the analytical continuation is done as a first step. In fact, in the standard case the energies of the fermions are defined in a discrete form through ip_n while in this new method the fermion energy is a continuous value $E+iI$. However, the difference of these two self-energies, Eqs. (20) and (E.3), can be shown to have a suitable form to relate it to the RPA response functions, that is,

$$\Sigma_{\rm ret}(\xi, E+iI) - \Sigma_{\rm qp}(\xi, E+iI) = \sum_{\eta}\sum_{\alpha,\rho\rho'} q^{(\alpha)}_{\rho\rho'}(\xi\eta)\left[1+n_B(E+iI-s^{(\alpha)}_{\xi\eta}E_\eta)-n_F(s^{(\alpha)}_{\xi\eta}E_\eta)\right] \times\sum_{q}\left(\frac{\mathcal{Q}^{(\alpha)*}_{\rho}(q)\,\mathcal{Q}^{(\alpha)}_{\rho'}(q)}{E+iI-s^{(\alpha)}_{\xi\eta}E_\eta-\omega^{(\alpha)}_q}-\frac{\mathcal{Q}^{(\alpha)*}_{\rho}(q)\,\mathcal{Q}^{(\alpha)}_{\rho'}(q)}{E+iI-s^{(\alpha)}_{\xi\eta}E_\eta+\omega^{(\alpha)}_q}\right). \tag{E.4}$$

Inserting Eq. (E.2) into Eq. (E.4), the retarded self-energy is thus expressed as

$$\Sigma_{\rm ret}(\xi, E+iI) = \sum_{\eta}\sum_{\alpha,\rho\rho'} q^{(\alpha)}_{\rho\rho'}(\xi\eta)\left[-\frac{1}{\beta}\sum_{i\omega_n}\left\{\frac{1}{E+iI-s^{(\alpha)}_{\xi\eta}E_\eta-i\omega_n} \times\sum_{q}\left(\frac{\mathcal{Q}^{(\alpha)*}_{\rho}(q)\,\mathcal{Q}^{(\alpha)}_{\rho'}(q)}{i\omega_n-\omega^{(\alpha)}_q}-\frac{\mathcal{Q}^{(\alpha)}_{\rho'}(q)\,\mathcal{Q}^{(\alpha)*}_{\rho}(q)}{i\omega_n+\omega^{(\alpha)}_q}\right)\right\} +\left[1+n_B(E+iI-s^{(\alpha)}_{\xi\eta}E_\eta)-n_F(s^{(\alpha)}_{\xi\eta}E_\eta)\right] \times\sum_{q}\left(\frac{\mathcal{Q}^{(\alpha)*}_{\rho}(q)\,\mathcal{Q}^{(\alpha)}_{\rho'}(q)}{E+iI-s^{(\alpha)}_{\xi\eta}E_\eta-\omega^{(\alpha)}_q}-\frac{\mathcal{Q}^{(\alpha)}_{\rho'}(q)\,\mathcal{Q}^{(\alpha)*}_{\rho}(q)}{E+iI-s^{(\alpha)}_{\xi\eta}E_\eta+\omega^{(\alpha)}_q}\right)\right]. \tag{E.5}$$

Using the formulae given in the next appendix, Eqs. (F.2)–(F.5), the summation of the RPA modes, $\sum_q$, can be performed, and the retarded self-energy can be written in terms of the response functions, according to

$$\begin{aligned}\Sigma_{\rm ret}(\xi, E+iI) = \sum_{\eta}\sum_{\alpha,\,\rho\rho'} q^{(\alpha)}_{\rho\rho'}(\xi\eta)\Bigg[&\frac{1}{\beta}\sum_{i\omega_n}\frac{1}{E+iI-s^{(\alpha)}_{\xi\eta}E_\eta - i\omega_n}\\ &\times\Bigg\{\tfrac{1}{2}\left(\mathcal{R}^{(\alpha)}_{\rho\rho'}(i\omega_n)-\mathcal{R}^{(\alpha)}_{\rho'\rho}(i\omega_n)\right)\\ &-\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{\mathcal{R}^{(\alpha)}_{\rho\rho'}(i\omega)-\mathcal{R}^{(\alpha)}_{\rho'\rho}(i\omega)}{i\omega_n - i\omega}\Bigg\}\\ &+\left[1+n_B(E+iI-s^{(\alpha)}_{\xi\eta}E_\eta)-n_F(s^{(\alpha)}_{\xi\eta}E_\eta)\right]\\ &\times\Bigg(\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{\mathcal{R}^{(\alpha)}_{\rho\rho'}(i\omega)-\mathcal{R}^{(\alpha)}_{\rho'\rho}(i\omega)}{E+iI-s^{(\alpha)}_{\xi\eta}E_\eta - i\omega}\\ &-\begin{cases}\mathcal{R}^{(\alpha)}_{\rho\rho'}(E+iI-s^{(\alpha)}_{\xi\eta}E_\eta), & \text{if } E>s^{(\alpha)}_{\xi\eta}E_\eta\\ \mathcal{R}^{(\alpha)}_{\rho'\rho}(E+iI-s^{(\alpha)}_{\xi\eta}E_\eta), & \text{if } E<s^{(\alpha)}_{\xi\eta}E_\eta\end{cases}\Bigg)\Bigg].\end{aligned} \tag{E.6}$$

This expression is further simplified by performing the frequency summation of the second term in the curly bracket such as

$$\begin{aligned}&\frac{1}{\beta}\sum_{i\omega_n}\frac{1}{E+iI-s^{(\alpha)}_{\xi\eta}E_\eta - i\omega_n}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{\mathcal{R}^{(\alpha)}_{\rho\rho'}(i\omega)-\mathcal{R}^{(\alpha)}_{\rho'\rho}(i\omega)}{i\omega_n - i\omega}\\ &\quad=\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\left[n_B(E+iI-s^{(\alpha)}_{\xi\eta}E_\eta)-n_B(i\omega)\right]\frac{\mathcal{R}^{(\alpha)}_{\rho\rho'}(i\omega)-\mathcal{R}^{(\alpha)}_{\rho'\rho}(i\omega)}{E+iI-s^{(\alpha)}_{\xi\eta}E_\eta - i\omega}.\end{aligned}$$

The final result is given in Eq. (23).

Appendix F. Integrals of the response function

We need two different integral forms to solve our problem, that is,

$$\begin{aligned}I^{(\alpha)}_{\rho\rho'}(E+iI) &= \int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{\mathcal{R}^{(\alpha)}_{\rho\rho'}(i\omega)}{E+iI-i\omega},\\ J^{(\alpha)}_{\rho\rho'}(i\omega_n) &= \int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{\mathcal{R}^{(\alpha)}_{\rho\rho'}(i\omega)}{i\omega_n - i\omega}.\end{aligned} \tag{F.1}$$

The important results that we obtain using these integrals are

$$\sum_q \frac{\mathcal{Q}_\rho^{(\alpha)*}(q)\mathcal{Q}_{\rho'}^{(\alpha)}(q)}{E+iI-\omega_q^{(\alpha)}}$$

$$= \begin{cases} \displaystyle\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\mathcal{R}_{\rho\rho'}^{(\alpha)}(i\omega)}{E+iI-i\omega} - \mathcal{R}_{\rho\rho'}^{(\alpha)}(E+iI)\,, & \text{if } E>0\,, \\ \displaystyle\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\mathcal{R}_{\rho\rho'}^{(\alpha)}(i\omega)}{E+iI-i\omega}\,, & \text{if } E<0\,, \end{cases} \tag{F.2}$$

$$\sum_q \frac{\mathcal{Q}_\rho^{(\alpha)}(q)\mathcal{Q}_{\rho'}^{(\alpha)*}(q)}{E+iI+\omega_q^{(\alpha)}}$$

$$= \begin{cases} \displaystyle\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\mathcal{R}_{\rho\rho'}^{(\alpha)}(i\omega)}{E+iI-i\omega}\,, & \text{if } E>0\,, \\ \displaystyle\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\mathcal{R}_{\rho\rho'}^{(\alpha)}(i\omega)}{E+iI-i\omega} + \mathcal{R}_{\rho\rho'}^{(\alpha)}(E+iI)\,, & \text{if } E<0\,, \end{cases} \tag{F.3}$$

$$\sum_q \frac{\mathcal{Q}_\rho^{(\alpha)*}(q)\mathcal{Q}_{\rho'}^{(\alpha)}(q)}{i\omega_n-\omega_q^{(\alpha)}} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\mathcal{R}_{\rho\rho'}^{(\alpha)}(i\omega)}{i\omega_n-i\omega} - \tfrac{1}{2}\mathcal{R}_{\rho\rho'}^{(\alpha)}(i\omega_n)\,, \tag{F.4}$$

$$\sum_q \frac{\mathcal{Q}_\rho^{(\alpha)}(q)\mathcal{Q}_{\rho'}^{(\alpha)*}(q)}{i\omega_n+\omega_q^{(\alpha)}} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\mathcal{R}_{\rho\rho'}^{(\alpha)}(i\omega)}{i\omega_n-i\omega} + \tfrac{1}{2}\mathcal{R}_{\rho\rho'}^{(\alpha)}(i\omega_n)\,. \tag{F.5}$$

References

[1] G.F. Bertsch, P.F. Bortignon, R.A. Broglia, Rev. Mod. Phys 55 (1983) 287.
[2] C. Mahaux, P.F. Bortignon, R.A. Broglia, C.H. Dasso, Phys. Rep. 120 (1985) 1.
[3] R.A. Broglia, V. Paar, D.R. Bes, Phys. Lett. 37B (1971) 59, 257.
[4] A. Bohr, B.R. Mottelson, Nulear Structure, Vol. II (Benjamin, Reading, MA, 1975).
[5] F. Barranco, R.A. Broglia, E. Vigezzi, P.F. Bortignon, J. Terasaki, E. Vigezzi, Phys. Rev. Lett., to be pubblished.
[6] P.F. Bortignon, R.A. Broglia, G.F. Bertsch, J. Pacheco, Nucl. Phys. A 460 (1986) 149.
[7] P. Donati, P.M. Pizzochero, P.F. Bortignon, R.A. Broglia, Phys. Rev. Lett 72 (1994) 2835.
[8] P. Donati et al., Z. Phys. A 354 (1996) 249.
[9] P.F. Bortignon, A. Bracco, R.A. Broglia, Giant Resonances; Nuclear Structure at Finite Temperature (Harwood Academic Publishers, New York, 1998).
[10] D.R. Bes, R.A. Broglia, Nucl. Phys. A 80 (1966) 289.
[11] F. Barranco, M. Gallardo, R.A. Broglia, Phys. Lett. B 198 (1987) 19.
[12] Y.R. Shimizu, J.D. Garrett, R.A. Broglia, M. Gallardo, E. Vigezzi, Rev. Mod. Phys. 61 (1989) 131.
[13] P.F. Bortignon, R.A. Broglia, D.R. Bes, R. Liotta, Phys. Rep. C 30 (1977) 30.
[14] D.R. Bes, J. Kurchan, The Treatment of Collective Coordinates in Many-Body Systems (World Scientific, Singapore, 1990).
[15] P. Donati, T. Døssing, Y.R. Shimizu, P.F. Bortignon, R.A. Broglia, Nucl. Phys. A 653 (1999) 27.
[16] P. Donati, T. Døssing, Y.R. Shimizu, P.F. Bortignon, R.A. Broglia, Nucleon–phonon coupling in hot, rapidly rotating nuclei (II) – Applications, to be published.

[17] R. Kubo, J. Phys. Soc. Jpn 17 (1962) 975.
[18] J.A.A.J. Perenboom, P. Wyler, F. Meier, Phys. Rep. 78 (1981) 173.
[19] P.W. Anderson, Basic Notion of Condensed Matter Physics (Benjamin, Menlo Park, 1984).
[20] P.W. Anderson, The Theory of Superconductivity in the High-T_c Cuprates (Princeton Univ. Press, Princeton, 1997).
[21] G. Mahan, Many Particle Physics (Plenum Press, New York, 1981).
[22] S.G. Nilsson, Mat. Fys. Medd., Dan. Vid. Selsk. 29 (1955) No. 16.
[23] Y.R. Shimizu, K. Matsuyanagi, Prog. Theor. Phys. 70 (1983) 144.
[24] M. Baranger, K. Kumar, Nucl. Phys. 62 (1965) 113.
[25] T. Kishimoto et al., Phys. Rev. Lett. 35 (1975) 552.
[26] H. Kurasawa, Prog. Theor. Phys. 64 (1980) 2055.
[27] T. Suzuki, D.J. Rowe, Nucl. Phys. A 289 (1977) 461.
[28] H. Sakamoto, T. Kishimoto, Nucl. Phys. A 501 (1989) 205.
[29] H.M. Sommermann, Ann. Phys. 11 (1983) 163.
[30] P. Ring, L.M. Robledo, J.L. Egido, M. Faber, Nucl. Phys. A 419 (1984) 261;
O. Civitarese, R.A. Broglia, C.H. Dasso, Ann. Phys. 156 (1984) 142;
H. Sagawa, G.F. Bertsch, Phys. Lett. B 146 (1984) 138.
[31] M. Matsuzaki, Y.R. Shimizu, K. Matsuyanagi, Prog. Theor. Phys. 79 (1988) 863.
[32] P. Donati, Self-Energies of Single-Particle States in Deformed Nuclei Generated by Dynamical Coupling to the Surface, Ph.D Thesis (July 1998).
[33] A. Fetter, J. Walecka, Quantum Theory of Many-Particle Systems (McGraw Hill, New York, 1971).
[34] A.A. Abrikosov, L.P. Gorkov, I.E. Dzyaloshinski, Quantum Field Theory in Statistical Physics (Prentice Hall, New York, 1963).
[35] A.L. Goodman, Nucl. Phys. A 230 (1974) 466;
J.W. Negele, E. Vogt, eds., Advances in Nuclear Physics, Vol. 11 (Plenum Press, New York, 1979) p. 263.
[36] T. Nakatsukasa, K. Matsuyanagi, S. Mizutori, Y.R. Shimizu, Phys. Rev. C 53 (1996) 2213.
[37] P. Ring, P. Schuck, The Nuclear Many-Body Problem (Springer, New York, 1980).