Temperature Dependence of the Nucleon Effective Mass and the Physics of Stellar Collapse

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The temperature dependence of the nucleon effective mass is calculated for the nuclei ^{98}Mo, ^{64}Zn, and ^{64}Ni. In all three cases, the effective mass is found to decrease appreciably in the temperature interval 0–1 MeV. Such a dependence has consequences, among other things, on the level-density parameter and on the symmetry energy. In particular, the effect of the increased symmetry energy on the neutronization processes in collapsing stars is estimated. We find indication that the larger initial lepton fraction thus obtained may significantly help the subsequent supernova explosion.

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The binding energy of atomic nuclei displays a strong dependence on the number of particles, in much the same way as the ionization potential of atoms. Hartree-Fock theory provides, as in the atomic case, the theoretical framework to calculate the mean field where the nucleons move. Because of the Pauli principle, the Hartree-Fock potential is nonlocal in the spatial coordinates. It is possible to absorb this nonlocality through an effective mass, the so-called "k mass" m_k. It is found that the k mass is smaller than the free nucleon mass m ($m_k \approx 0.7m$ [1]).

Consequently, static mean field theory is able to provide the correct sequence of single-particle levels around the Fermi energy, but fails in reproducing the observed level density. This is because the level-density parameter is proportional to the *total* nucleon effective mass (cf. Ref. [2]).

The solution to this problem has been found by recognizing that the mean field is not static, but highly dynamic. Among the modes associated with the fluctuations of this field one finds vibrations of the nuclear surface. The coupling of the fermions to the phonons describing these modes leads to self-energy corrections which are nonlocal in time, since the phonons propagate in the nuclear medium with finite velocity. Also, here, this nonlocality can be absorbed in terms of an effective mass, the so-called "ω mass" defined as $m_\omega = m[1-\partial\tilde{V}(\omega)/\partial\omega]$, where $\tilde{V}(\omega)$ is the Fourier transform of the (time) nonlocal part of the potential (cf. Ref. [3] and references therein). Dressed particles are heavier than the bare nucleon, the net result being that the total effective mass is $m^* = m_k m_\omega / m \approx m$, as required by the experimental findings.

While this picture is deceptively similar to that associated with the motion of nucleons in an empirical Woods-Saxon potential and carrying bare nucleon mass, important differences are also found, in particular, in the temperature dependence of quantities which depend on the effective mass. The energy of low-lying collective vibrations (related to the ω mass) is of the order of 1–3 MeV, while the rigidity of the mean field (related to the k mass) is controlled by the energy difference between major shells $\hbar\omega_0 = 41A^{-1/3}$ MeV, which is ≈ 8 MeV for medium heavy nuclei. Consequently, the question of the temperature dependence of the total effective mass is tantamount to the temperature dependence of the ω mass, at least within the range of temperature under discussion ($0 \le T \le 3$ MeV). It should be noted that there has been a large activity in the calculation of the renormalization of the single-particle energies at finite temperature. Most of it, however, was done in connection with the calculation of the damping width of giant resonances. Concerning the calculation of the ω mass at finite temperature, essentially only two groups have been active [4–6].

In the present work we study this dependence, with special reference to the symmetry energy, that is, the energy needed to separate protons from neutrons. The calculation was carried out for the nuclei $^{98}_{42}$Mo$_{56}$, $^{64}_{30}$Zn$_{34}$, and $^{64}_{28}$Ni$_{36}$ of astrophysical interest. In the Matsubara formalism (cf., e.g., Ref. [7]), the self-energy correction to the Hartree-Fock field can be expressed in terms of the coupling of single-particle states to the collective vibrations according to [4,5]

$$\Sigma_{(2)}(1,\omega+iI,T) = \sum_{2,\lambda} V^2(1,2;\lambda)\left(\frac{1+n_B(\lambda)-n_F(2)}{\omega+iI-\epsilon_2-\omega_\lambda} + \frac{n_B(\lambda)+n_F(2)}{\omega+iI-\epsilon_2+\omega_\lambda}\right). \tag{1}$$

The labels 1 and 2 are the quantum numbers of the single-particle states involved in the process, with energies ϵ_1 and ϵ_2, respectively. The collective vibration is denoted by λ, the associated energy by ω_λ. The quantity $V(1,2;\lambda)$ is the matrix element with which particles 1 and 2 couple to the vibration. The quantities

$$n_B(\lambda) = \left[\exp\left(\frac{\omega_\lambda}{T}\right)-1\right]^{-1}$$

and

$$n_F(i) = \left[\exp\left(\frac{\epsilon_i-\epsilon_F}{T}\right)+1\right]^{-1}$$

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are the Bose-Einstein and Fermi occupation numbers, respectively, ϵ_F being the Fermi energy and T the temperature. Finally, the interval of energy over which averages are taken is indicated by I.

The potential appearing in the definition of m_ω is related to $\Sigma_{(2)}$ according to $\tilde{V}(\omega)=\mathrm{Re}\Sigma_{(2)}$ [3]. This quantity is quite subtle, as it is controlled by a myriad of "off-the-energy-shell" contributions and consequently by a proper treatment of an eventual cutoff. The calculation of Eq. (1) was carried out making use of a single-particle basis provided by empirical potentials of the Woods-Saxon type, of the form $U(r)=U_0/\{1+\exp[(r-R)/a]\}$. Allowing the radius to oscillate, one obtains in first order in the deformation parameter $\alpha_{\lambda\mu}$ the expression $\delta U=-R_0 \times[\partial U(r)/\partial r]\sum_{\lambda\mu}\alpha_{\lambda\mu}Y^*_{\lambda\mu}(\hat{\mathbf{r}})$. This potential provides a coupling between the single-particle motion and the collective vibrations of the surface. Making use of this potential and of the single-particle energies and wave functions, one can calculate the properties of the collective modes. This has been done within the framework of linear response theory, in the so-called quasiparticle random phase approximation (QRPA) [2].

The existence at $T=0$ MeV of low-lying states carrying a sizable fraction of the energy-weighted sum rule (EWSR) is apparent from our results. In particular the lowest 2^+ state of ^{98}Mo carries 10% of the EWSR and an associated $B(E2)$ value corresponding to 21.3 Weisskopf units (W.u.), while the lowest 3^- carries 14% of the EWSR and 33 W.u. These are the states which play the most important role in the renormalization process leading to the ω mass. The corresponding values of this quantity (as collected in Table I) testify to the fact that the couplings involved do renormalize the single-particle motion in an important way. Increasing the temperature of ^{98}Mo to $T=2$ MeV has no consequences for the high-lying energy regime ($\gtrsim 10$ MeV), corresponding to the region of the giant resonances, while significant changes are observed at low excitation energies ($\lesssim 3$ MeV), where temperature essentially obliterates the correlations. In the case of the lowest 3^- state mentioned above, the reduction of collectivity amounts to 50%. Similar results are obtained for the other nuclei under discussion.

One can thus expect a reduction, with temperature, of the ω mass. This is indeed the case as can be seen from Table I. The increase of the ω mass over the bare mass value, as measured by the quantity $-\partial\tilde{V}(\omega)/\partial\omega$, is reduced by $\sim 50\%$ already within the first MeV of temperature. For later use, it is convenient to give an analytical form to the temperature dependence of m_ω. The exponential formula $m_\omega(T)=m+[m_\omega(0)-m]\exp(-T/T_0)$ turns out to be satisfying, and the best fitting values for the parameter T_0 are reported in the last column of Table I. In the calculation of Ref. [5] for the nucleus ^{208}Pb, a larger value of T_0 was found, of the order of 3.3 MeV. This is consistent with the higher frequency of the vibrational states in ^{208}Pb, the lowest state being at 2.6 MeV. An experimental consequence of the reduction of the ω mass with temperature has been detected in the lowering of the value of the level-density parameter ($a\sim m^*$) needed to reproduce the decay spectra in heavy ion reactions at intermediate energies (cf. Ref. [8] and references therein).

TABLE I. Values of the ratio $m_\omega(T)/m$ as a function of the temperature for three different nuclei. The last column gives the best value of the parameter T_0 for the exponential fit $m_\omega(T)=m+[m_\omega(0)-m]\exp(-T/T_0)$.

	$T=0$ MeV	$T=1$ MeV	$T=2$ MeV	T_0 (Mev)
^{98}Mo	1.7	1.36	1.26	1.89
^{64}Zn	1.8	1.45	1.3	1.97
^{64}Ni	1.45	1.2	1.2	2.05

We can now estimate the effect of the temperature dependence of the ω mass on the symmetry energy. This quantity appears in the nuclear equation of state as $E_S=s(0)(1-2x)^2$, where $x=Z/A$ is the concentration of protons in nuclei of mass A and the symmetry coefficient is $s(0)\approx 30$ MeV at $T=0$ [9]. In the Fermi-gas model it can be written as the sum of a kinetic and a potential contribution, that is, $E_S=(E_S)_{\text{kin}}+(E_S)_{\text{pot}}$, where $(E_S)_{\text{kin}}$ scales as $1/m^*$ while $(E_S)_{\text{pot}}$ does not depend on m^*. In more realistic models the dependence of E_S on m^* is not known, but as a first approximation it is reasonable to take the same behavior; i.e., we assume that only the kinetic term depends on the effective mass. We then define the symmetry coefficient at finite temperature as $s(T)=s(0)+(\hbar^2k_0^2m/6m_k)[m_\omega(T)^{-1}-m_\omega(0)^{-1}]$, where we used the expression of $(E_S)_{\text{kin}}$ from Ref. [9] and the definition $m^*=m_km_\omega/m$, and where $k_0=1.35$ fm^{-1} corresponds to nuclear saturation density.

The change of E_S with temperature can now be found, remembering the fact that mean field quantities are essentially temperature independent, in any case within the range of temperature under discussion ($0\leq T\leq 3$ MeV). Therefore, assuming $m_k\approx 0.7m$ and making use of the values of Table I, one obtains an increase $\delta s\sim 2.5$ MeV in the temperature interval 0–1 MeV, corresponding to an increase in the symmetry energy of $\sim 8\%$. This change of the symmetry energy with temperature is expected to lead to an increase in the excitation energy of the giant dipole resonance [$\hbar\omega_{\text{GDR}}\sim s(T)^{1/2}$] which, in medium heavy nuclei and for a temperature of ≈ 3 MeV, is of the order of 0.7 MeV. Because of limited statistics, this prediction cannot be tested with presently available data, which are also consistent with a constant centroid within the experimental errors [10]. Specific experiments are being planned to remedy this situation [11].

We can now discuss the influence of the temperature dependence of m_ω on the physics of stellar collapse [12]. Here we shall just outline those points which are relevant to our discussion. When the "Fe" (neutron-rich iron-group nuclei) core of a massive star reaches its Chandrasekhar mass [13] and begins to collapse, the increasing density shifts the β equilibrium and induces electron capture on both free protons and protons bound in nuclei. As long as the density remains lower than a critical value, the trapping density $\rho_{\rm trap} \sim 5\times10^{11}$ g cm^{-3} [14], the neutrinos produced in the electron capture processes escape freely from the core. Therefore, these neutronization processes cause a net loss of lepton fraction Y_e [15] only in the range $\rho \leq \rho_{\rm trap}$. We shall denote by $\Delta Y_e = (Y_e)_i - (Y_e)_{\rm trap}$ the amount of deleptonization prior to shock formation, where typical initial conditions from stellar evolution calculations are $\rho_i \approx 4\times10^9$ g cm^{-3}, $T_i \approx 0.7$ MeV, and $(Y_e)_i \approx 0.42$.

It has long been known that a higher lepton fraction at trapping, i.e., a smaller value of ΔY_e, yields a stronger shock wave and can thus help the supernova explosion (in both the prompt and delayed mechanism scenario) [16]. The main reason for this is that the shock wave loses less energy in dissociating nuclei. Indeed, the shock forms at the edge of the so-called "homologous core," corresponding to the Chandrasekhar mass $M_{\rm Ch}[(Y_e)_{\rm trap}]$, and has to reach the edge of the "Fe" core, corresponding to the Chandrasekhar mass $M_{\rm Ch}[(Y_e)_i]$. During its propagation the shock dissociates the nuclei into α particles and neutrons, thus losing an enormous amount of energy. Since the "Fe" mass to traverse is $\Delta M = M_{\rm Ch}[(Y_e)_i] - M_{\rm Ch}[(Y_e)_{\rm trap}]$ and the dissociation energy for iron is ~ 17 foe per solar mass (1 foe $=10^{51}$ erg), the energy lost by the shock wave while getting out of the core is $E_{\rm diss} \approx 98[(Y_e)_i^2 - (Y_e)_{\rm trap}^2]$ (foe). It is now evident how a larger value for $(Y_e)_{\rm trap}$ implies a smaller loss of energy and thence a stronger shock wave.

At this point the physical connection between nucleon effective mass and stellar collapse can be done. We first point out that the rate of electron capture on both free and bound protons depends in a very sensitive way on the difference $\hat\mu = \mu_n - \mu_p$ between neutron and proton chemical potentials [17]. Larger values of $\hat\mu$ inhibit the neutronization process, since it becomes more difficult to transform a proton into a neutron. Now, as the density increases from ρ_i to $\rho_{\rm trap}$, the temperature varies in the range 0.7–1.5 MeV, while the typical equilibrium nuclear species present in the collapsing core have A in the region $\sim$55–100 and Z in the region $\sim$25–40 (which explains our choice of nuclei to study). Since the drop of the nucleon effective mass in the first MeV of temperature yields a sizable increase in the symmetry energy, and since $\hat\mu \propto E_S + \cdots$, where the extra terms do not depend on m^*, we conclude that the temperature dependence of the ω mass will quench the neutronization processes in the initial phase of the collapse, i.e., will give smaller values for ΔY_e.

In order to estimate the size of the effects just described, we have followed analytically the gravitational collapse of the core in a one-zone model along the lines of Ref. [17] (cf. also [18]). More precisely, the hydrodynamical acceleration equation is replaced by a "one-zone" collapse equation for the density change, while the temperature evolution is derived from the entropy changes due to the energy balance between escaping neutrinos and collapsing matter. Although schematic, the model implements in a satisfactory way several physical features of the problem: (i) Nuclei are described by the mass formula of Ref. [9] with bulk, surface, and Coulomb terms, and with momentum dependence included. Such an equation of state was developed to describe matter in neutron stars and it should apply well to the neutron-rich nuclei present in the collapse. (ii) Neutron drip from nuclei and dissociation of nuclei into neutrons and alphas are taken into account. It turns out that α dissociation is actually negligible, while the free neutron fraction X_n increases from $\sim 10^{-3}$ at ρ_i to $\sim 10^{-1}$ at $\rho_{\rm trap}$. (iii) Entropy terms for all the degrees of freedom are included. The internal nuclear entropy is treated in the Fermi-gas model as in Ref. [17], with corrections due to the finite nuclear surface introduced through the effective mass. (iv) Electron capture on both free and bound protons is included along the lines of Ref. [17]. The detailed treatment of Gamow-Teller transitions of Ref. [18] was taken into account only schematically, by introducing a quenching factor $\gamma^2 = 0.1$ for the capture on protons bound in nuclei.

The nucleon effective mass enters the model through two quantities (and their derivatives [19]): The symmetry energy of the nuclear EOS and the nuclear entropy. The results of our calculation are given in Fig. 1. They are reported in terms of the parameters $m_\omega(0)$ and T_0 of the exponential fit for $m_\omega(T)$. Although a very wide region of values was studied, Table I shows that the range of physical interest is $1.4m < m_\omega(0) < 1.8m$ and $1.9 < T_0 < 2.1$ MeV. In the graph we plot the *gain* in dissociation energy $\delta E_{\rm diss}$ obtained when letting the ω mass depend on temperature instead of keeping it constant, i.e.,

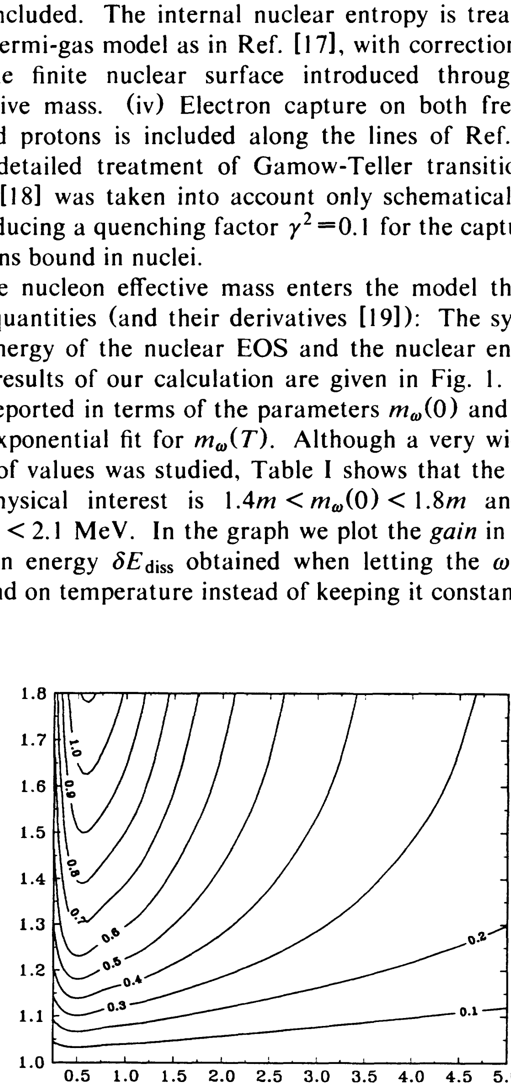

FIG. 1. Gain in dissociation energy between the cases with temperature-dependent and constant ω mass, as a function of the parameters $m_\omega(0)/m$ and T_0 of the exponential fit. Contour level curves with a spacing of 0.1 foe are plotted.

$m_\omega(T)=m_\omega(0)$.

The general features of Fig. 1 can be understood with the following simple heuristic argument. Since the quantity $\delta E_{\rm diss}$ represents the *difference* between the cases with temperature-dependent and constant m_ω, it must depend *mainly* on the temperature derivative of $m_\omega(T)$ evaluated at some temperature T_c in the physical range, typically $T_c \approx T_i = 0.7$ MeV. This derivative goes like $[m_\omega(0)-m]/T_0 \exp(-T_c/T_0)$, and this expression (or any monotonically increasing function of it) accounts for all the features of $\delta E_{\rm diss}$ in the plot. On the one hand, indeed, at fixed T_0 it increases monotonically with $m_\omega(0)$, starting at zero when $m_\omega(0)/m=1$. On the other hand, at constant $m_\omega(0)$ it increases from zero (not seen in the plot, which is cut at $T_0=0.25$ MeV) up to a maximum at $T_0=T_c \approx T_i$, and then decreases monotonically for larger values of the parameter T_0. Asymptotically, for $T_0 \rightarrow \infty$ the temperature dependence is lost and no effect is seen.

From Fig. 1 we conclude that, in the range of physical interest, the temperature dependence of the effective mass yields a larger value for the lepton fraction at trapping, with a decrease of $\sim$(7–9)% in the amount of deleptonization ΔY_e. In turn this corresponds to a gain in dissociation energy of $\sim$0.5–0.6 foe. This number is of the same order of magnitude as the typical explosion energy of a type II supernova (for the case of SN 1987A, a value $E_{\rm expl} \sim 1$ foe was found), so that the outcome of the explosions obtained in detailed numerical simulations could be significantly changed by taking this nuclear effect into account.

Of course, a simple one-zone treatment suffers from deficiencies, since it neglects the presence of density and temperature gradients along the core and their physical consequences (cf. also [14]). We have, however, performed several sensitivity tests: (i) trying different values for the initial conditions ρ_i, T_i, and $(Y_e)_i$; (ii) increasing the trapping density $\rho_{\rm trap}$; and (iii) dropping the momentum dependence of the nuclear equation of state. We have found that the results change numerically [for reasonable test values, $\delta E_{\rm diss}$ can vary up to (20–30)%] but their order of magnitude is not affected.

The limitations of the above calculation of the effective mass stem from two sources. The first one is that the calculations are not self-consistent. The second one is associated with the assumption that the mean field is spherically symmetric. At finite temperatures, this ansatz is not correct, as the system will assume all possible deformations with a probability controlled by the associated Boltzmann factors. While the first effect is not deemed important, the second one has been studied in Ref. [8] and found to be negligible as well. One should also be aware that conclusions based on schematic supernova models can be significantly changed by nonlinear effects and feedback mechanisms in realistic numerical models. Furthermore, the dependence on temperature of the other terms of the nuclear EOS, which do not depend on m^*, should be introduced into the picture.

In any case, we believe that the temperature dependence of the nucleon effective mass must have an appreciable effect on the physics of stellar collapse, and that any physically satisfactory model should take such a dependence into account.

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